Correct definition of the Poynting vector in electrically and magnetically polarizable medium reveals that negative refraction is impossible: reply

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Abstract: In this reply I argue that “the work done on the bound current densities” can not be “attributed differently”, as Favaro, Kinsler and McCall (FKM) suggest in the preceding Comment. There is a unique physically justifiable expression for this quantity, namely, $\mathbf{J}_b \cdot \mathbf{E}$. This expression follows from the Newtonian laws of motion. Alternative forms can not be obtained by mathematically manipulating the Poynting identity. I also discuss other points raised by FKM in their Comment.

OCIS codes: (160.1245) Artificially engineered materials; (350.3618) Left-handed materials

References and links

1. A. Favaro, P. Kinsler, and M. W. McCall, “Comment on “Correct definition of the Poynting vector in electrically and magnetically polarizable medium reveals that negative refraction is impossible”,” Opt. Express **17**(17), 15167–15169 (2009).
2. V. A. Markel, “Correct definition of the Poynting vector in electrically and magnetically polarizable medium reveals that negative refraction is impossible,” Opt. Express **16**, 19152–19168 (2008).

In the concluding part of their Comment [1], Favaro, Kinsler and McCall (FKM) write: ”In summary, we have shown that whilst $\mathbf{E} \times \mathbf{B}$ can be a useful choice of Poynting vector for some [physical situations? types of media? - V.M.] (see e.g. the recent [7]), it is no more or less correct than $\mathbf{E} \times \mathbf{H}$.” This conclusion is hardly acceptable for a reasonable physicist. Even if one believes (as I do) that the Poynting vector itself is not experimentally measurable, its divergence certainly is an observable physical quantity and the two definitions given above have different divergences. How can FKM state that there are two different formulas for the same observable quantity which yield different results but are nevertheless both simultaneously correct? Moreover, how can a mathematical expression be correct to varying degrees? It is either correct (within a certain theoretical frame) or not. If FKM think that the first form is applicable to one type of media and the second form to another, they should at least state the physical conditions of applicability of each expression. Ref.7 of the Comment, which FKM have mentioned, is rather generic and does not consider different cases. FKM also point the reader to Ref.3 of the Comment in which, they claim, it is shown that ”the two continuity equations describe energy conservation and the material response in different but complementary ways”. However, as of this writing, this paper is still unpublished. Therefore, I will discuss only the content of the Comment itself. In what follows, I will explain the logical mistake that FKM have made in

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arriving at their unusual conclusion and address other less significant points raised by FKM. Equations of the Comment are referenced below as (C#).

The main logical flaw of the FKM's argument is that FKM think that the Poynting vector can be defined arbitrarily by manipulating the Poynting identity. According to FKM, whatever expression appears inside the divergence term (written explicitly as a divergence), is the Poynting vector. If in Eq. (C2) the expression $\mathbf{E}\times\mathbf{B}$ appears inside the divergence, it is one possible form of the Poynting vector. If in Eq. (C7) $\mathbf{E}\times\mathbf{H}$ appears inside the divergence, this is another valid choice which is "no more or less correct" than the first one. Of course, it is possible to define an infinite number of Poynting vectors in this manner by splitting different parts of the quantity $\nabla\cdot(\mathbf{E}\times\mathbf{B})$, assigning new notations to them, rewriting Eq. (C2) in new variables, and calling whatever is left inside the divergence term the new Poynting vector.

Therefore, it appears to me that FKM have misunderstood the main point of Ref. [2]. This point was that the work which is done by the electric field on bound currents induced in the medium must be computed from first principles and can not be deduced from Maxwell's equations or from the Poynting identity which is an elementary consequence of the latter. And the Poynting vector also can not be deduced from Maxwell's equations (which have no work or energy) but must be chosen so that it is compatible with the expression for the work done on the charges.

Let me explain this in more detail. First, there is really no doubt that Eqs. (C2) and (C7) are proper identities. For that matter, Eqs. (C8) and (C9), (C10) and (C11) are also identities. However the difference is that Eqs. (C2),(C8) and (C10) contain the term $\mathbf{J}_b\cdot\mathbf{E}$, as FKM say, *explicitly*. If we believe that $\mathbf{J}_b\cdot\mathbf{E}$ is the work done by the electric field on moving charges (and this seems to follow from the Newtonian mechanics), then this is the quantity that must be computed to determine the heating rate. In a stationary process which I have considered in Ref. [2], there is simply no other channel for this work to disappear apart from irreversible conversion to heat (either positive or negative). Thus, I could have avoided discussing of the Poynting vector altogether. I could have simply computed the quantity $\langle\mathbf{J}_b\cdot\mathbf{E}\rangle$ where $\langle\ldots\rangle$ stands for time averaging. In fact, I did so in my paper and the result was that the heating rate in an isotropic local medium (inside the volume) is proportional to $\mathrm{Im}[\varepsilon(\omega)\mu(\omega)]$.

Of course, I felt a need to explain the discrepancy with the conventional result and that is why I have endeavored to discuss the correct form of the Poynting vector. Yet, from the fundamental point of view, this discussion was not necessary. All relevant results obtain directly from evaluating the expression $\langle\mathbf{J}_b\cdot\mathbf{E}\rangle$. However, if the Poynting vector is introduced, it must, at least, satisfy energy conservation. In a stationary process when the potential energy density does not change on average, the energy conservation law has the form $\langle\nabla\cdot\mathbf{S}\rangle+\langle\mathbf{J}_b\cdot\mathbf{E}\rangle=0$. Any expression for the Poynting vector must satisfy this condition.

Now FKM may or may not believe that, in a stationary process, the work $\langle\mathbf{J}_b\cdot\mathbf{E}\rangle$ goes entirely into heat or that the energy conservation is expressed as $\langle\nabla\cdot\mathbf{S}\rangle+\langle\mathbf{J}_b\cdot\mathbf{E}\rangle=0$. It would be helpful if FKM had stated their position regarding this question clearly in the Comment, but this was not done. Instead, FKM wrote "In Eq. (C2) this work on the bound current density is expressed explicitly as $\mathbf{J}_b\cdot\mathbf{E}$ but in Eq. (7) it is implicit, being included via $\mathbf{P}$ and $\mathbf{M}$ and the definitions of $\mathbf{D}$ and $\mathbf{H}$". What this statement means is not clear to me. Sure, one can split an arbitrary part from the total work $\mathbf{J}_b\cdot\mathbf{E}$ which stands as the last term in the l.h.s. of Eq. (C2) and group it together with other terms in the same equation and the identity will not cease to hold. But the important point is that the new terms that result from this manipulation have no clear physical interpretation. Whereas we can say that it follows from Newtonian mechanics and the expression for the Lorentz force that $\langle\mathbf{J}_b\cdot\mathbf{E}\rangle$ is work done on bound charges, what can we say, for example, about the term $\mathbf{E}\cdot\partial_t\mathbf{E}+\mathbf{H}\cdot\partial_t\mathbf{B}$? Why should we interpret it in any particular way?

There needs to be a basis for a physical interpretation of various terms in the Poynting iden-

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tity and the only such basis is provided by the expressions for the Lorentz force and by the laws of classical mechanics. Any interpretation that is not based on these two principles is arbitrary and unprovable. Therefore, the difference between, say, Eqs. (C10) and (C11) is apparent: in (C10) the l.h.s. can be physically interpreted as work while in (C11) the l.h.s. has no first-principles physical interpretation. FKM write correctly that "...the discrepancy between the RHSs of Eqs. (C8) and (C9), as revealed through Eqs. (C10) and (C11), clearly cannot be used to establish the correctness of $\mathbf{E}\times\mathbf{B}$ over $\mathbf{E}\times\mathbf{H}$ ... for the Poynting vector." But I did not employ such a circular logic in my paper. Of course, I understand and have always understood that equalities (C8-C10), although different in form, are all correct. However, I have argued that only in Eqs. (C8) and (C10) the terms can be invested with physical meaning while (C9) and (C11) are formal identities which should not be interpreted in any particular way. All this is discussed in Ref. [2] on pp.19158-19159 and FKM have apparently either missed or misunderstood this discussion.

FKM also take an issue with my definition of negative refraction which is based on the inequality $\mathrm{Im}(\varepsilon\mu) < 0$. I have applied this condition to local, isotropic, passive media. For more general media, a more general criterion was given in Sec. 6 of Ref. [2] (after Eq. 55). In any case, I have stated clearly that the condition $\mathrm{Im}(\varepsilon\mu) < 0$ applies only to passive media and that in active media the inequality must be reversed, with which FKM seem to agree. The only point of disagreement here may be the definition of active and passive media. By active media I have meant the media in which the refracted wave increases in amplitude away from the interface while in passive media the amplitude must decrease. If there is no energy input to the medium, it is obviously passive and the condition $\mathrm{Im}(\varepsilon\mu) < 0$ fully applies, assuming there is no anisotropy or nonlocality. If, however, there is some energy input, the situation is more complicated. The supplied energy may be insufficient to overcome losses, so that the medium is still passive according to my definition. I did not discuss these nuances in my paper and distinguished only between two possibilities: the field amplitude either decays or grows away from the interface.

Finally, the FKM's comment regarding the conventional definition of negative phase velocity relative to the direction of the Poynting vector is irrelevant. The condition $\mathrm{Im}(\varepsilon\mu) < 0$ can be used to determine whether a plane wave would refract at a negative angle at a planar interface irrespectively of the definition of the Poynting vector. All that is needed is the proper boundary conditions at infinity. For passive media, the boundary conditions state that the electric and magnetic fields should vanish very far from the interface. Generally speaking, the knowledge of the Poynting vector is not needed to solve Maxwell's equation and to determine which angle with the interface the refracted wave should make.

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