Unidirectional Wave in Structured Chain Waveguides

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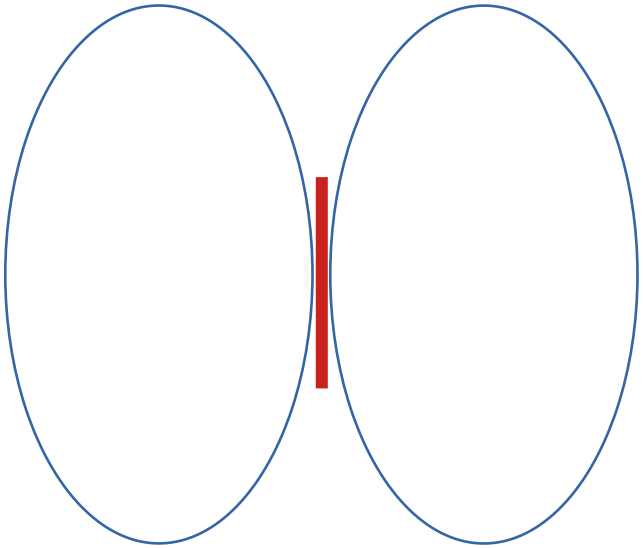
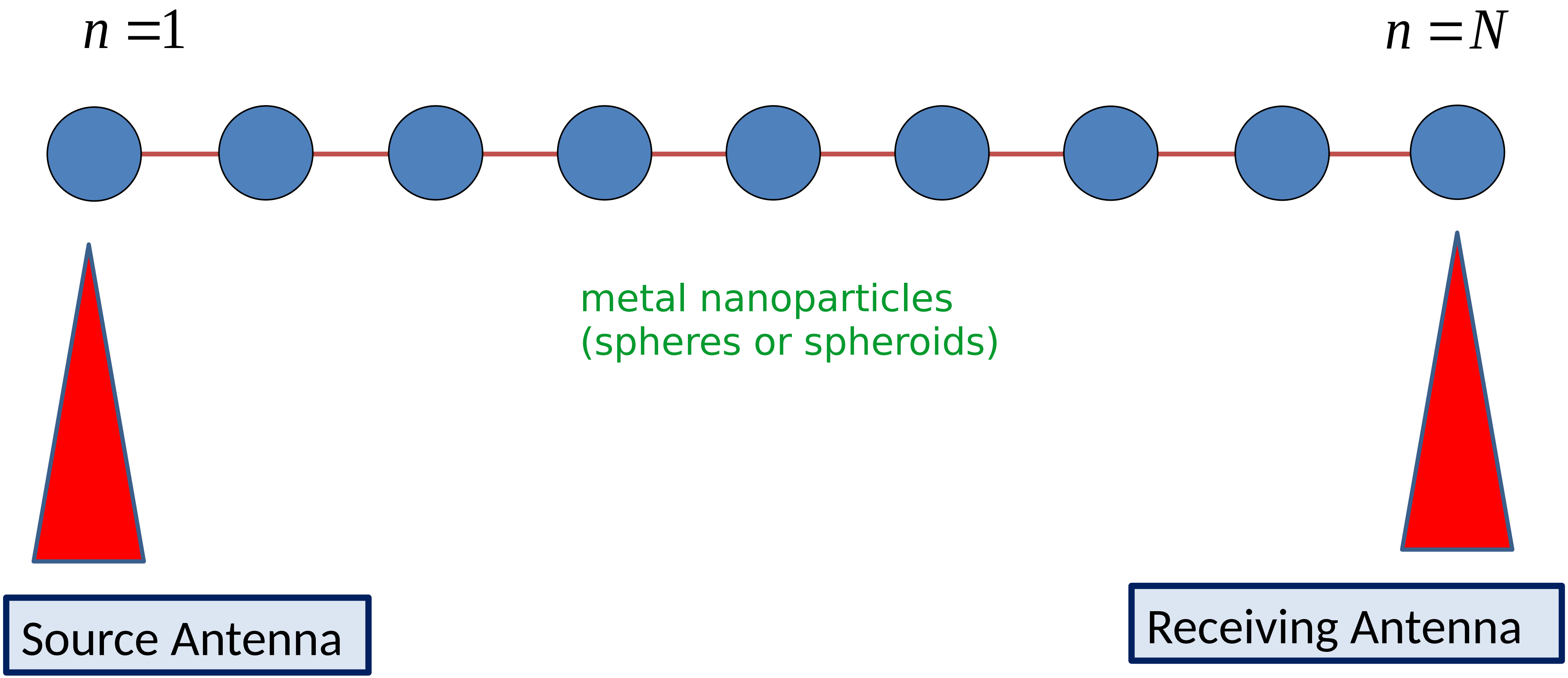
Electrically small antenna

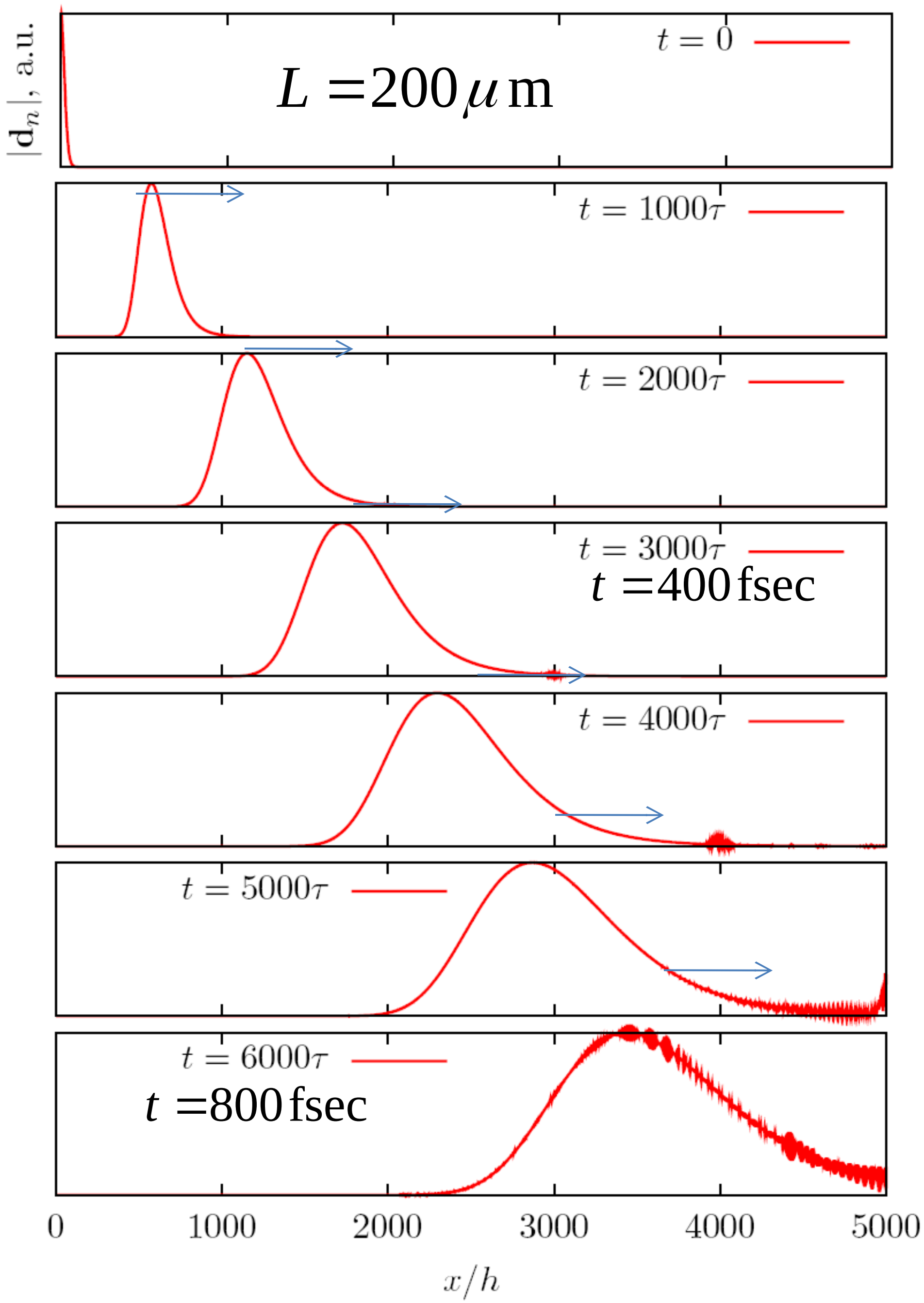


A small antenna in vacuum cannot create a collimated beam

A simple linear chain waveguide



Excitation can propagate from one end to another



Chain Parameters:

$h = 40\,\mathrm{nm}$

$b = 10\,\mathrm{nm}$

$\xi = \frac{b}{a} = 0.15$

$N = 5000$

$\tau = \frac{h}{c} = 0.133\,\mathrm{fsec}$

Metal Parameters (Ag)

$\varepsilon = \varepsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$

$\lambda_p = \frac{2\pi c}{\omega_p} = 136\,\mathrm{nm}$

$\gamma/\omega_p = 0.002$

$\varepsilon_0 = 5$

Host Medium:

$\varepsilon_h = 2.5$

$v_g \approx 0.58c$

Pulse Parameters:

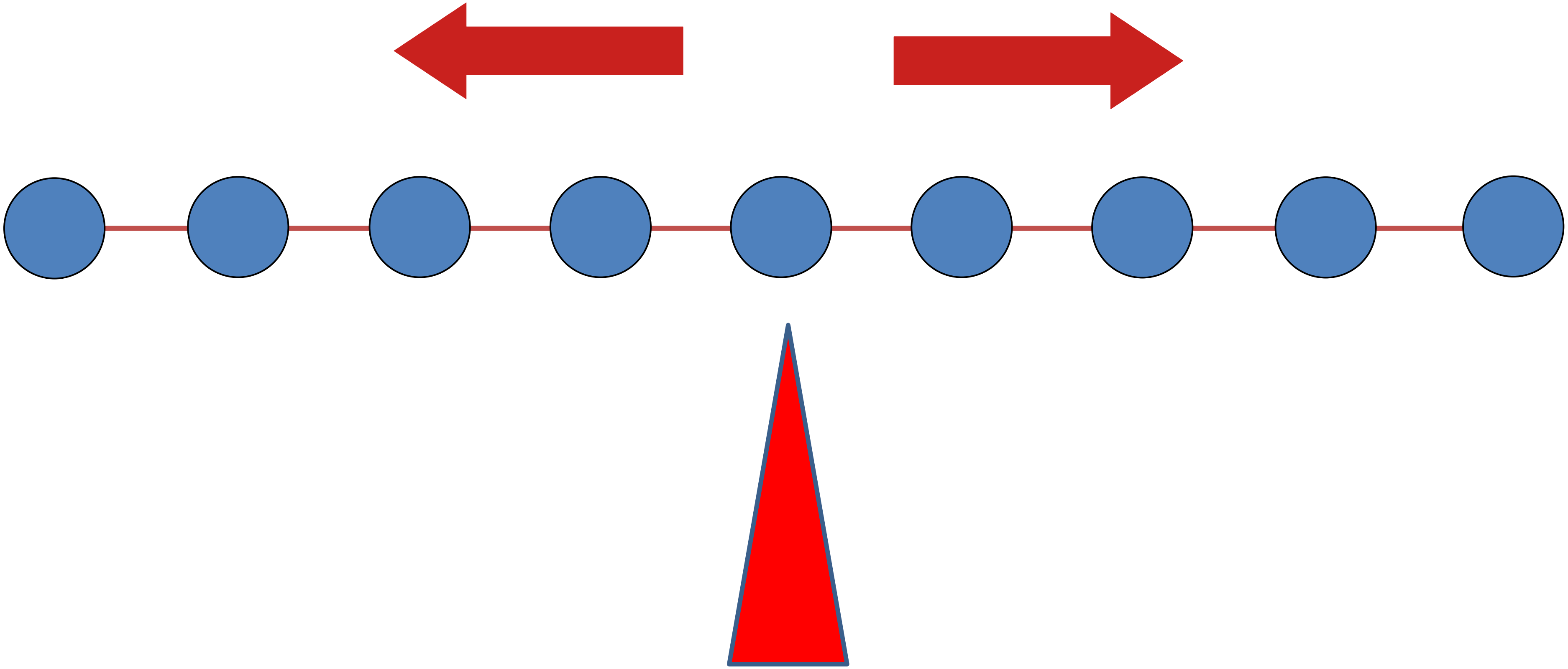
$\omega_0 = 0.1\omega_p$ [$\lambda_0 = 1.36\,\mu\mathrm{m}$]

$\Delta t = 7.2\,\mathrm{fsec}$

$\Delta\omega/\omega_0 = 0.2$

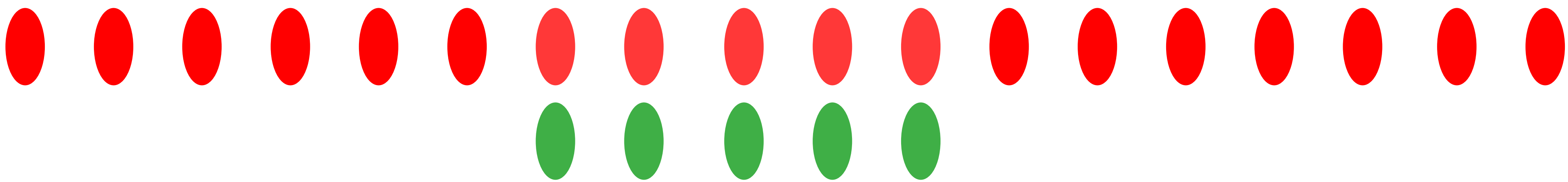
From A.A. Govyadinov, V.A. Markel, Phys. Rev. B 78 (3), 035403, 2008

What if we put the antenna in the middle?

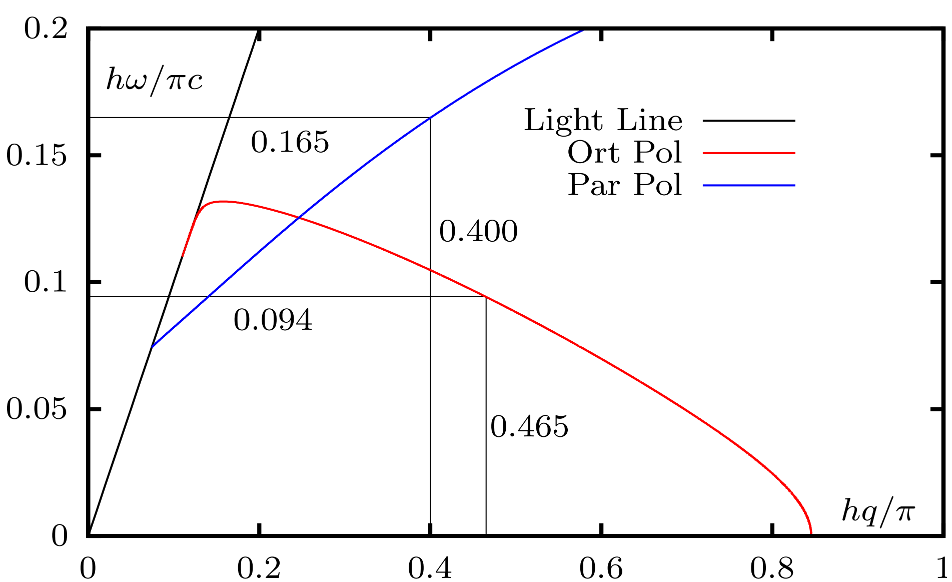


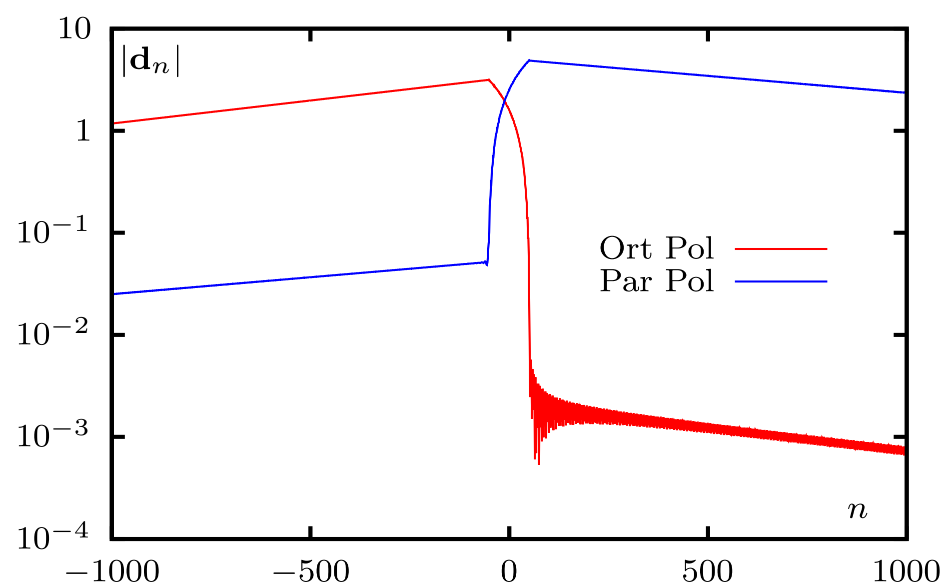
Excitation will propagate in both directions

What if we use a phased array antenna (not small)?

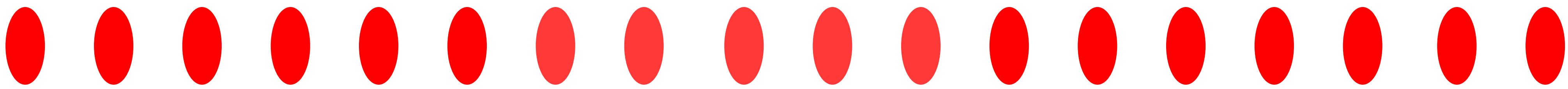


Phased array source is modulated with
the wave number q that is on the dispersion
curve of the waveguide at the working frequency

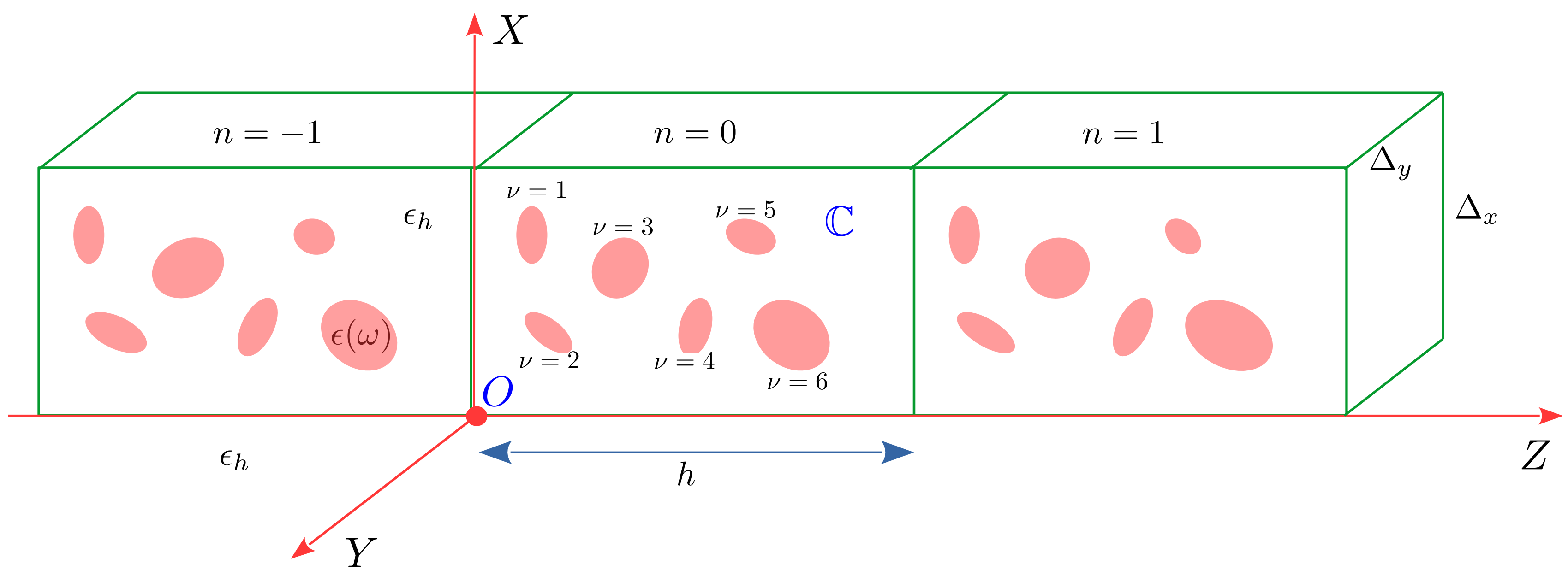




Can we make an **electrically small** antenna that will send energy in one direction only?



In a simple periodic chain like this one, this is impossible



But in a structured chain with a sense of direction, we can hope to see the effect

Chain must be periodic, or else it is not a waveguide!

The dispersion relation in a directional chain is still symmetric (from reciprocity)

$$\det[s(\omega)\hat{I} - \hat{W}(\omega, q)]$$

This is the dispersion equation

$$s(\omega) := \frac{\epsilon_h}{\epsilon(\omega) - \epsilon_h}$$

This is the spectral parameter of the theory (the only term that depends on the material of particles)

for a unit cell containing p arbitrary ellipsoids (all matrices are of size $3p \times 3p$):

$$\hat{W}(\omega, q) := \hat{B}\left[\ \hat{S}(\omega, q) + \frac{2k^3}{3}\hat{I}\ \right] - \hat{K}$$

- $\hat{B}$: Diagonal matrix containing volumes
- $\hat{S}(\omega, q)$: Dipole sum
- $\frac{2k^3}{3}\hat{I}$: Radiative correction
- $\hat{K}$: Matrix contains depolarization coefficients and rotation angles

Reciprocity of Green's function

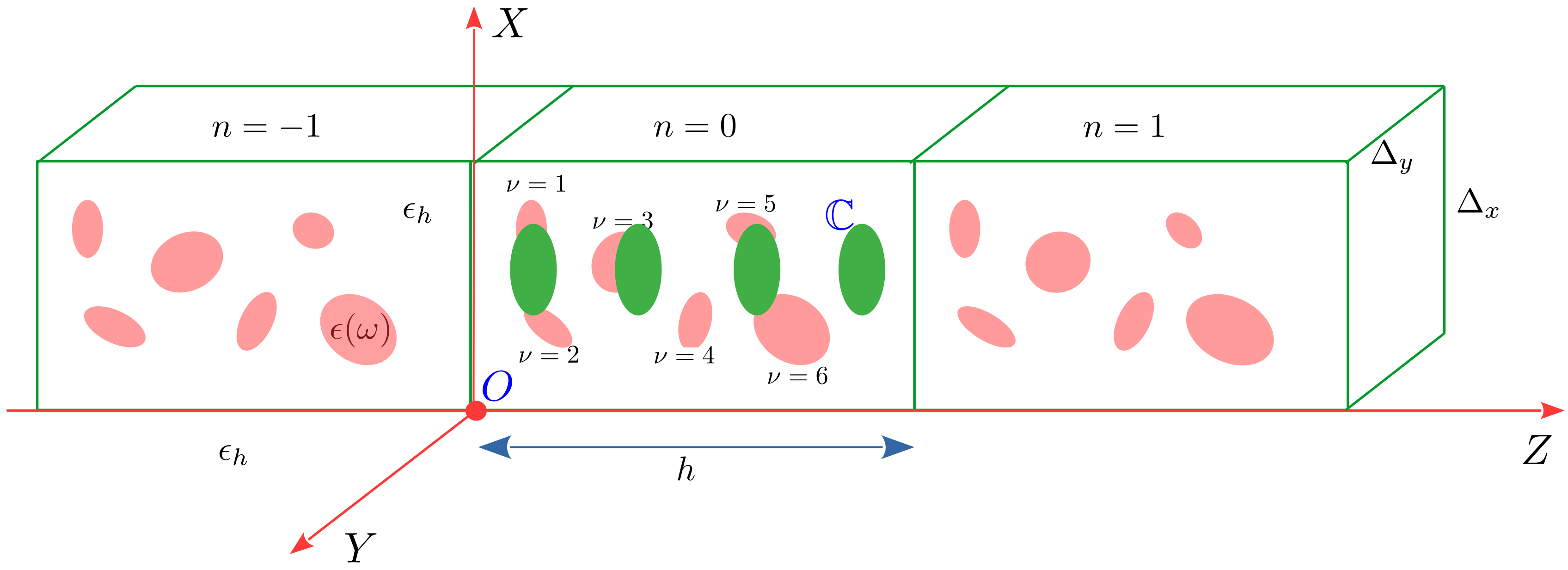
↓

$$\hat{S}^T(\omega, q) = \hat{S}(\omega, -q)$$

↓

$\hat{W}(\omega, q)$ and $\hat{W}(\omega, -q)$ share the same eigenvalues

We can make an **electrically small** antenna confined to one unit cell that sends energy in one direction only



In empty space, this antenna still radiates as a dipole (because the unit cell is electrically small)

In a directional chain, the effect is possible because

$$\hat{W}(\omega, q) \neq \hat{W}^T(\omega, q)$$

We can understand why unidirectional propagation is possible if *W* is not symmetric from the quasi-particle pole approximation

$$|d_n\rangle = \text{const} \int_{-\pi/h}^{\pi/h} \left[s(\omega)\, \hat{I} - \hat{W}(\omega, \xi) \right]^{-1} |e\rangle\, e^{\mathrm{i}\,\xi h n}\, \mathrm{d}\xi \ .$$

Vector (length 3*p*) of dipole moments in *n*-th cell

Vector of external fields, which are assumed to be localized to 0-th cell

$$\det \left[s(\omega)\, \hat{I} - \hat{W}(\omega, q) \right] = 0$$

This equation determines the dispersion relation $q=q(\omega)$ moments in *n*-th cell

We now use spectral properties of the matrix *W*

For simplicity, let $\hat{B} = \beta \hat{I}$ (all ellipsoids are of the same volume)

$$\hat{W}(\omega, -q) = \hat{W}^T(\omega, q)$$

$$\hat{W}(\omega, q)\,|f_i(\omega, q)\rangle = \lambda_i(\omega, q)\,|f_i(\omega, q)\rangle$$

$$\langle f_i(\omega, -q)|\,\hat{W}(\omega, q) = \langle f_i(\omega, -q)|\,\lambda_i(\omega, q)$$

$$\langle f_i(\omega, -q)\,|\,f_j(\omega, q)|\rangle = \delta_{ij}\,Z_i\ , \qquad Z_i \neq 0$$

$$\hat{W}(\omega, q) = \sum_{i=1}^{3p} \frac{1}{Z_i(\omega, q)}\ \lambda_i(\omega, q)\,|f_i(\omega, q)\rangle\,\langle f_i(\omega, -q)|$$

Let only one eigenvalue be at resonance at the working frequency

$$|d_n\rangle = \mathrm{const} \int\limits_{-\pi/h}^{\pi/h} \mathrm{d}\xi \, e^{\mathrm{i}\xi h n} \, Z_r(\omega,\xi) \, \frac{|f_r(\omega,\xi)\rangle \, \langle f_r(\omega,-\xi)|e\rangle}{s(\omega) - \lambda_r(\omega,\xi)}$$

⬇

$$|d_n\rangle = \mathrm{const} \, e^{[\mathrm{i}q(\omega)-\gamma(\omega)]h|n|} \begin{cases} |d_+\rangle \, , & n > 0 \\ \dfrac{|d_+\rangle \, + \, |d_-\rangle}{2} \, , & n = 0 \\ |d_-\rangle \, , & n < 0 \end{cases}$$

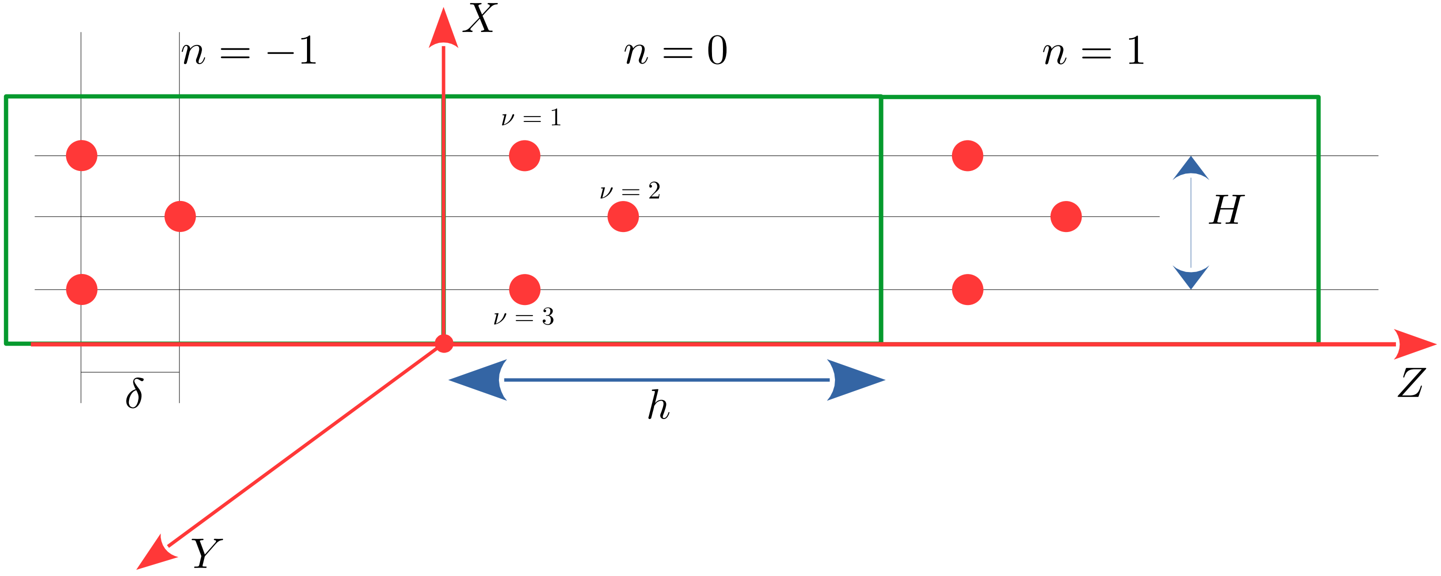
$$|d_+\rangle = |f_r(\omega,q)\rangle\langle f_r(\omega,-q)|e\rangle$$

$$|d_-\rangle = |f_r(\omega,-q)\rangle\langle f_r(\omega,q)|e\rangle$$

The trick of unidirectional coupling is to make such |e> that one vector is zero but the other is not.

In directional chains this is possible.

Example: Simplest directional chain (prolate spheroids viewed from top)



Polarization is out of plane of drawing

Algebraic properties of W

$$\hat{W} = \begin{bmatrix} a & b & c \\ d & a & d \\ c & b & a \end{bmatrix}$$

This is the algebraic structure of W:
it is neither symmetric nor Hermitian

$$\lambda_1 = a - c \,, \quad \lambda_2 = a + \frac{c - \sqrt{c^2 + 8bd}}{2} \,, \quad \lambda_3 = a + \frac{c + \sqrt{c^2 + 8bd}}{2}$$

Eigenvalues

$$|f_1\rangle = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$|f_2\rangle = \begin{bmatrix} 1 & \dfrac{-\sqrt{c^2 + 8bd} - c}{2b} & 1 \end{bmatrix}$$

$$|f_3\rangle = \begin{bmatrix} 1 & \dfrac{\sqrt{c^2 + 8bd} - c}{2b} & 1 \end{bmatrix}$$

Right eigenvectors

$$|g_1\rangle = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$|g_2\rangle = \begin{bmatrix} 1 & \dfrac{-\sqrt{c^2 + 8bd} - c}{2d} & 1 \end{bmatrix}$$

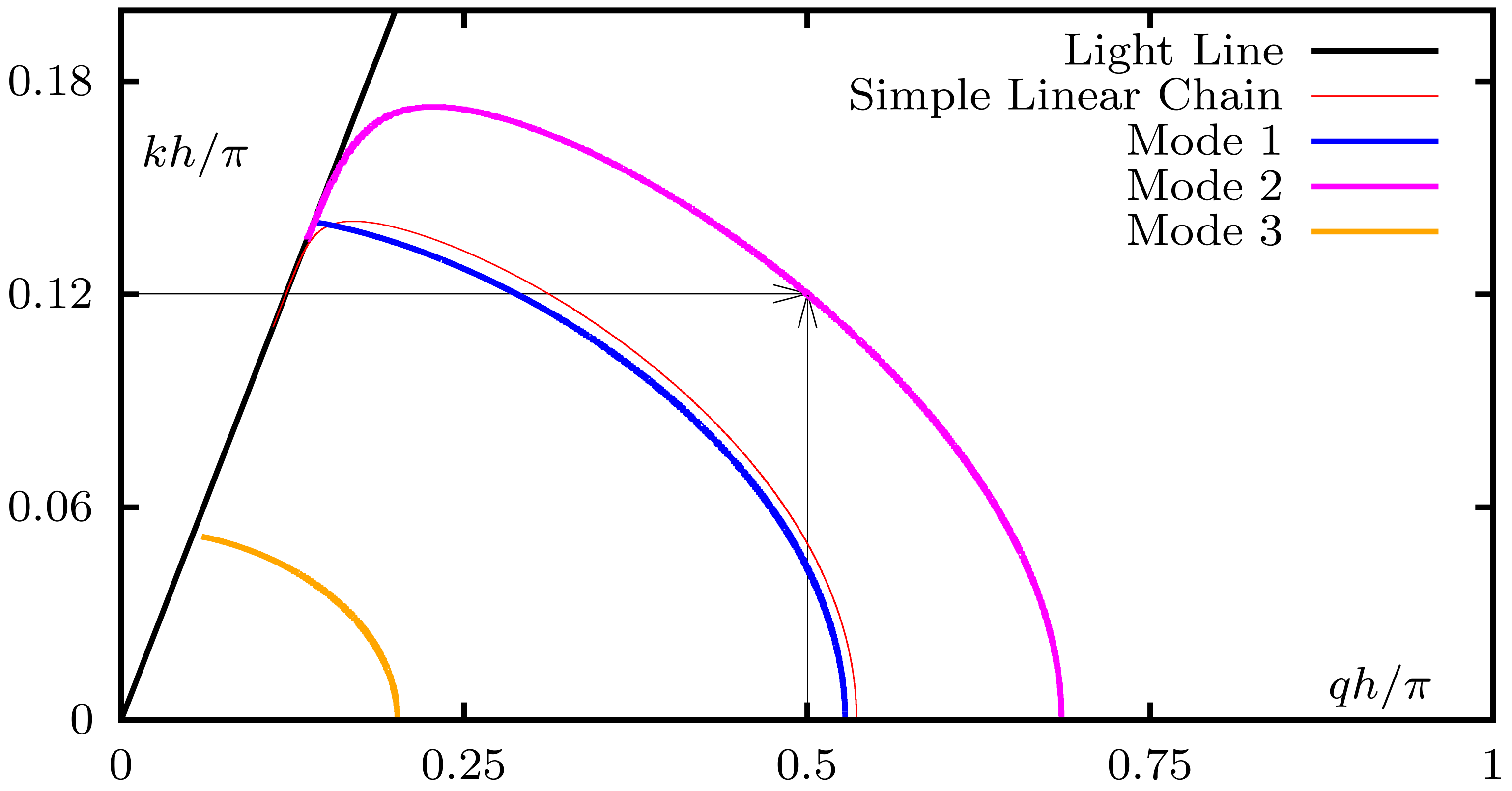
$$|g_3\rangle = \begin{bmatrix} 1 & \dfrac{\sqrt{c^2 + 8bd} - c}{2d} & 1 \end{bmatrix}$$

Left eigenvectors

$$|d_+\rangle = |f_2\rangle\langle g_2|e\rangle \,, \quad |d_-\rangle = |g_2\rangle\langle f_2|e\rangle$$

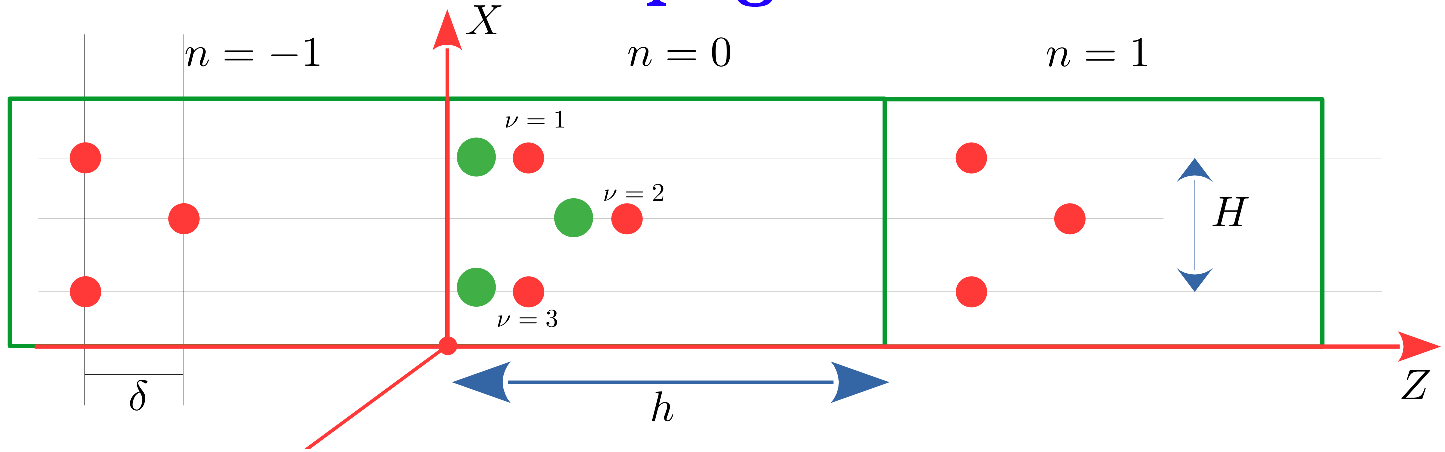
Assuming 2^{nd} mode is in resonance

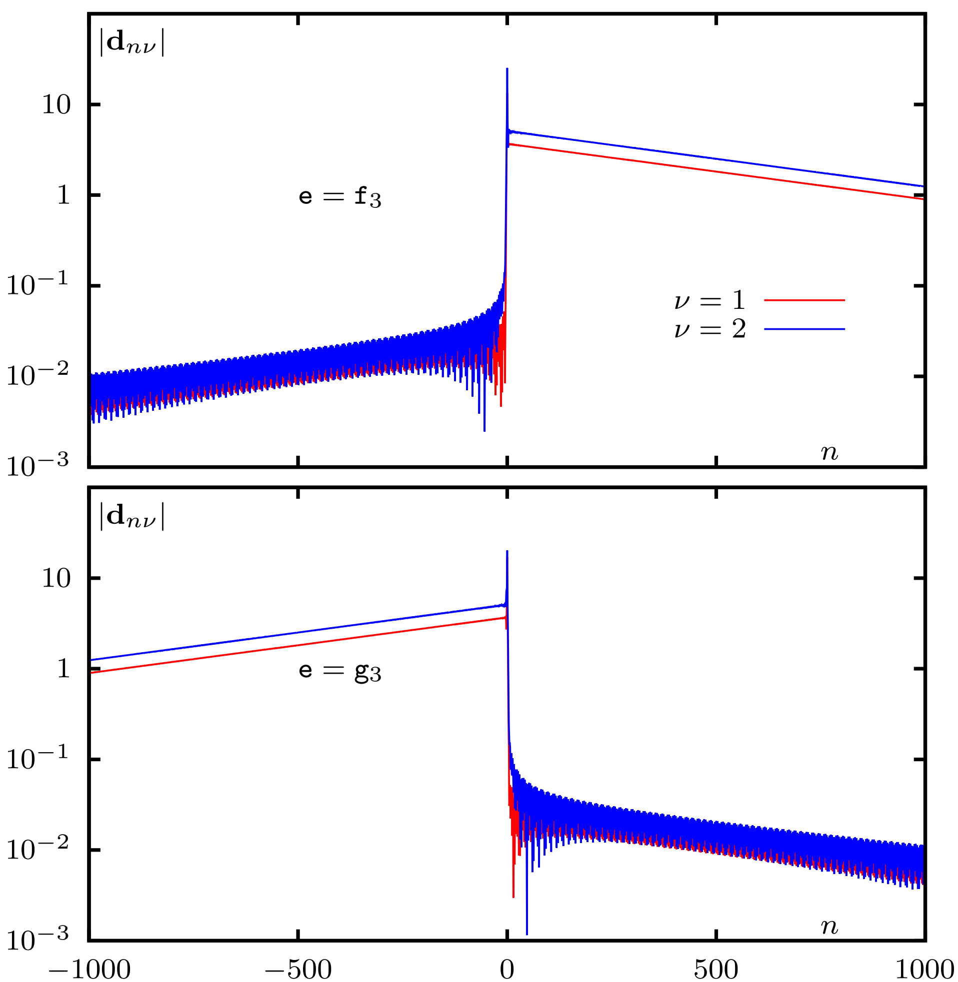
Dispersion relation



Simulation for prolate spheroids;
h=25.3nm, *H*=2*h*=50.6nm, *a*=6.325nm, *b*=42.17nm
Drude metal with eps_0=5.0

Propagation





Choose these vectors for excitation:

$$|e\rangle = |f_3\rangle =$$

$$\begin{bmatrix} 1 & -(1.37131 + 0.471286\,\mathrm{i}) & 1 \end{bmatrix}$$

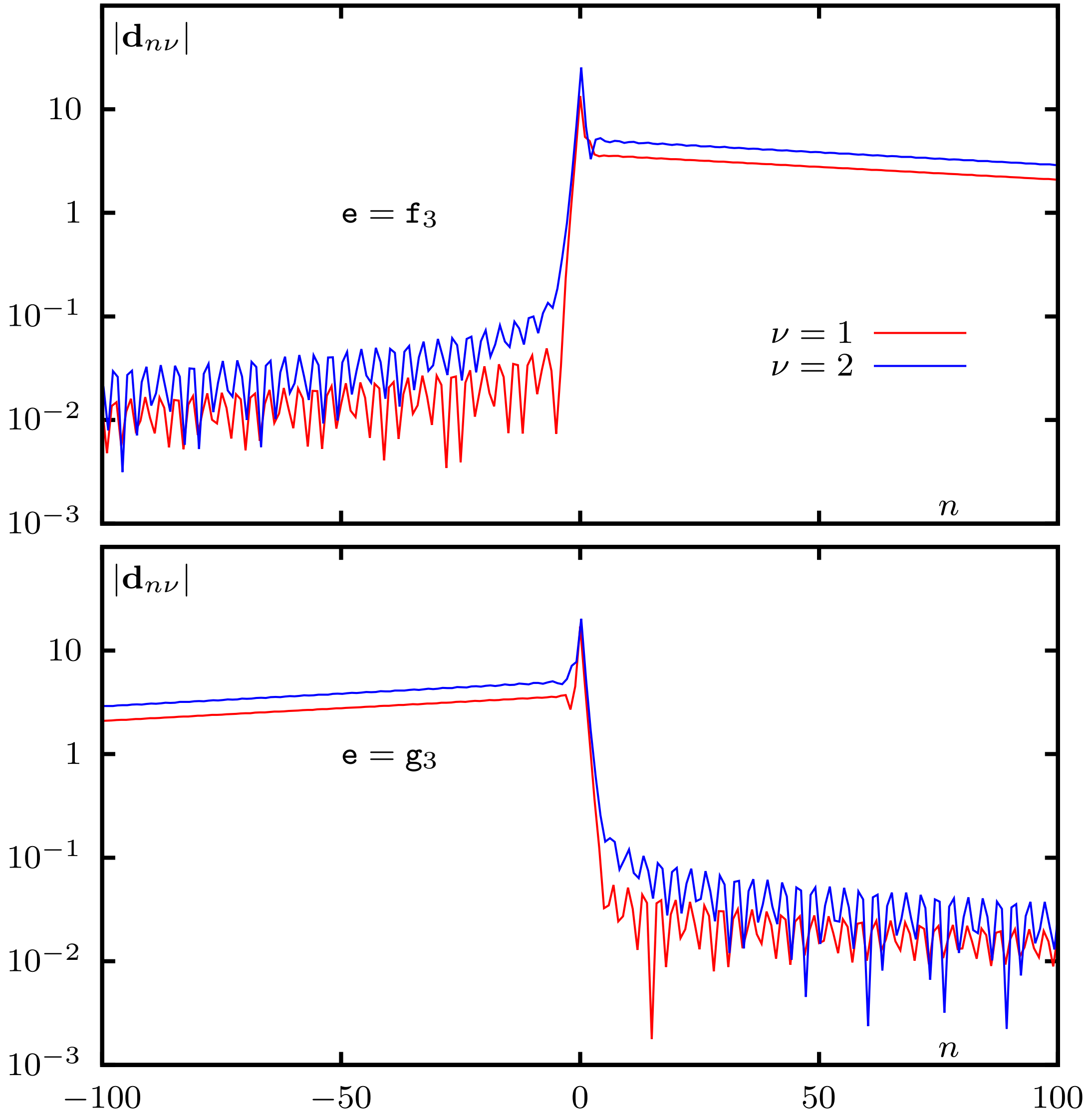
$$|e\rangle = |g_3\rangle =$$

$$\begin{bmatrix} 1 & -(1.37131 - 0.471286\,\mathrm{i}) & 1 \end{bmatrix}$$

Here we have used orthogonality:

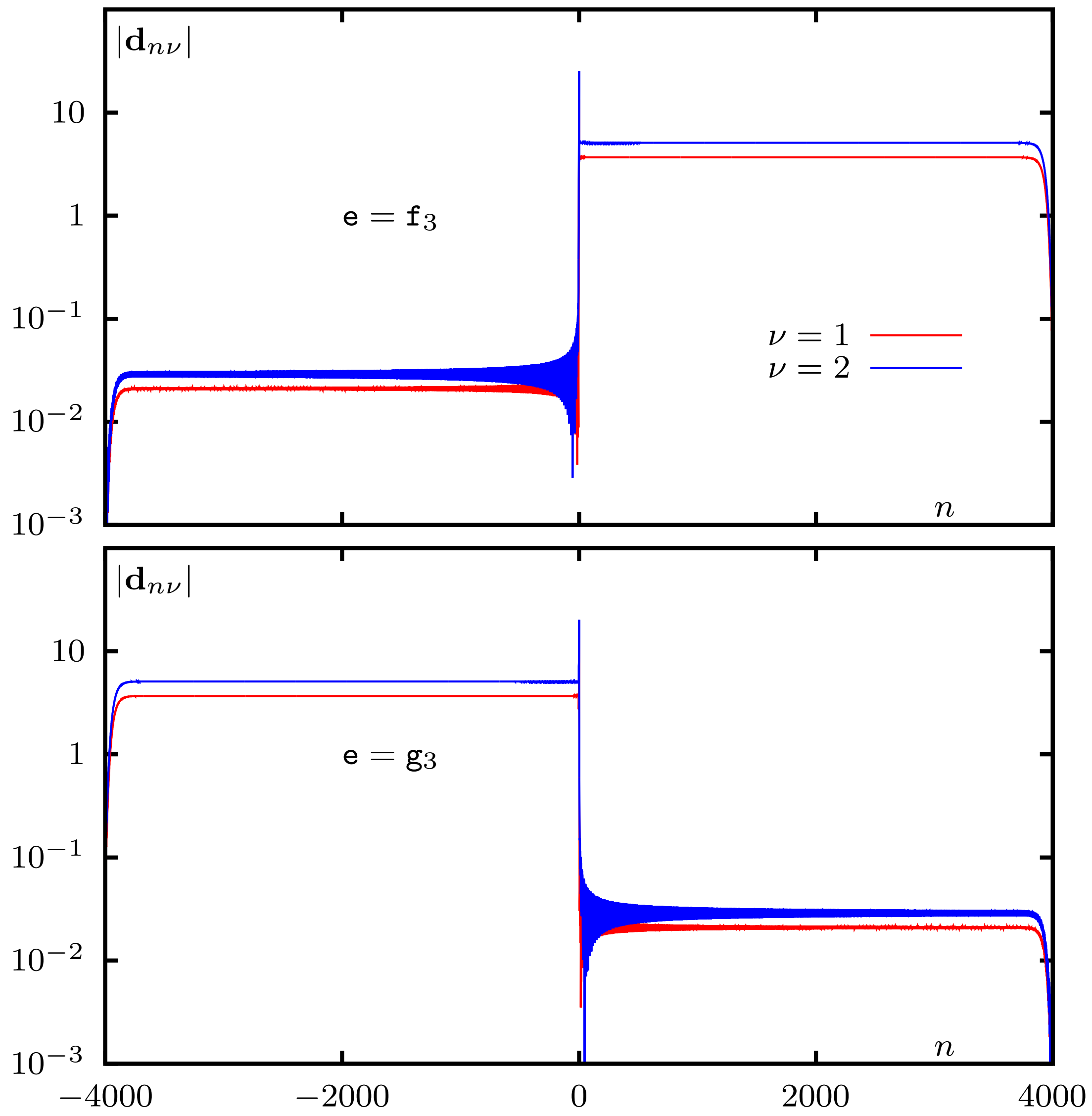
$$\langle f_2 | g_3 \rangle = \langle g_2 | f_3 \rangle = 0$$

Realistic losses



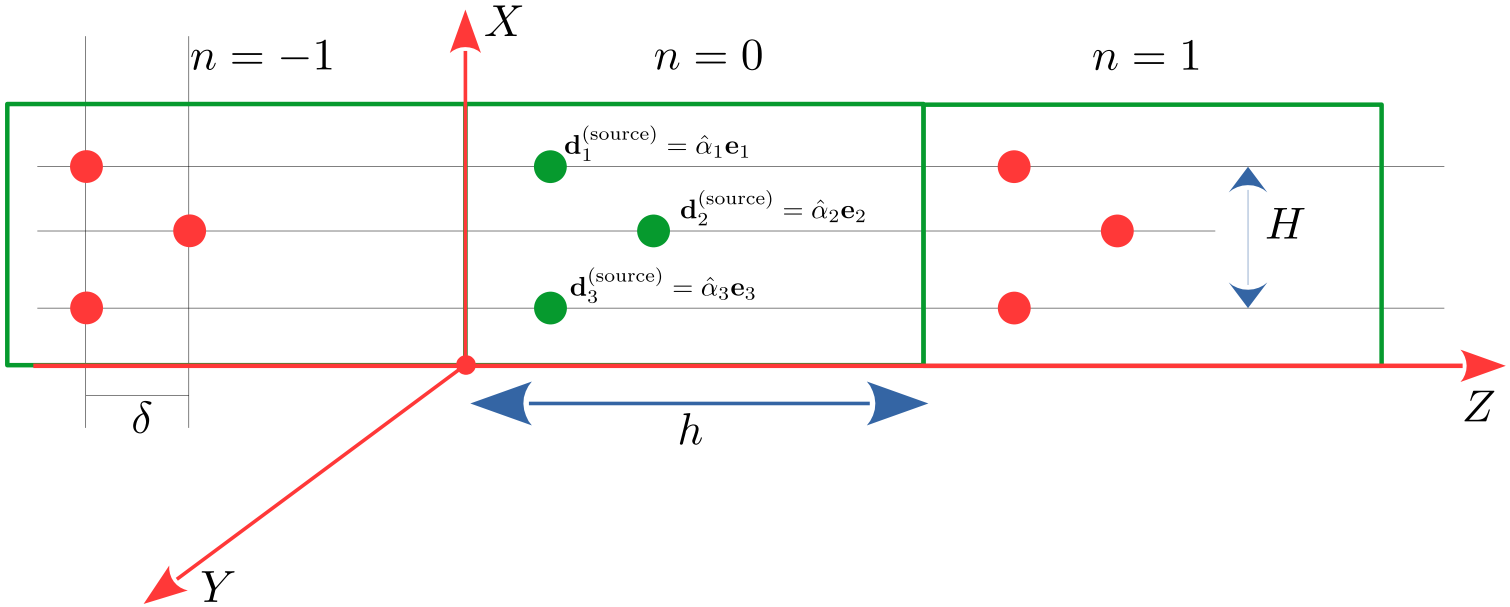
Here $\gamma/\omega_p = 0.002$ (as in silver)

Absorptive traps



Without absorptive traps there will be reflections from each end of the chain!

Chains with active segments



It is possible to show theoretically that the chain with active segment defined above produces the same excitation as in previous slides

Conclusions

- Unidirectional propagation due to an electrically small antenna is possible
- The modes are stable with respect to perturbations that preserve periodicity, but not to general defects!
- Defects produce back scattering
- There are also reflections from the chain ends; these can be suppressed by absorptive traps
- Will there be unidirectional propagation in simple chains made of non-reciprocal materials? I do not think so; directionality of the chain is required.
- But in non-reciprocal directional chains there will be no reflections from true defects or chain end (because there might be no back-propagating mode at the working frequency)
- So far I see this as the only way to suppress back-reflections