On Impossibility of Negative Refraction

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REFERENCES:

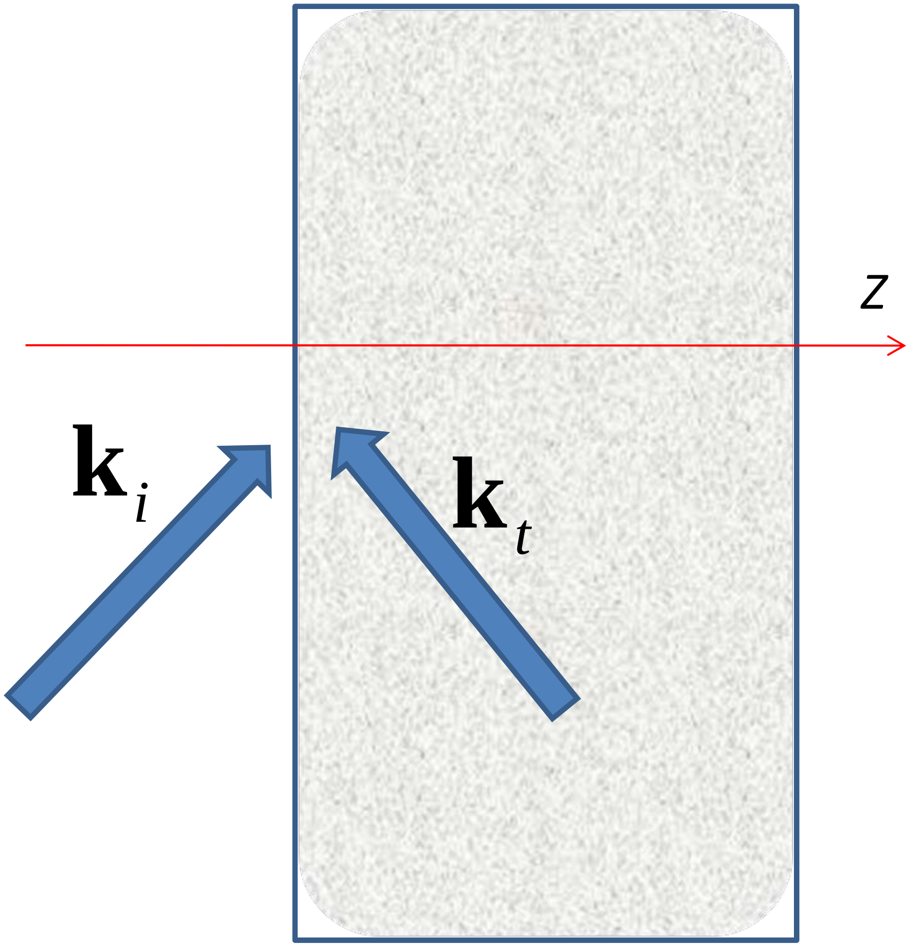
V.A.Markel, "Correct definition of the Poynting vector in electrically and magnetically polarizable media reveals that negative refraction is impossible," ***OpEx*** **16, 19152 (2008)**

Comment by Marques: *OpEx* 17, 7325 (2009)

Reply by Markel: *OpEx* 17, 7325 (2009)

What Do I Mean by Negative Refraction?

Refraction of a plane wave at a planar interface



NR OCCURS IF AND ONLY IF

$$\operatorname{Im}(\mathbf{k}\cdot\mathbf{k}) = \operatorname{Im}(k_{tz}^2)$$
$$= 2\operatorname{Re}(k_{tz})\operatorname{Im}(k_{tz}) < 0$$

In isotropic medium, this is equivalent to

$$\operatorname{Im}(\varepsilon\mu) < 0$$

I do not mean to say that deflection of a beam at a negative angle is not possible, e.g., in anisotropic medium.

But this phenomenon isn’t NR.

“Why do you say this? It looks like NR to me!”

I do not mean to say that negative dispersion or negative group velocity in photnonic crystals is not possible.

My results only apply to electromagnetically homogeneous media.

Heating Rate in Isotropic Media (Stationary Case)

$q = \langle \mathbf{J} \cdot \mathbf{E} \rangle_{\text{time}}$ - the heating rate

$\mathbf{J} = \partial \mathbf{P} / \partial t + c \nabla \times \mathbf{M}$ - the total current created by all charges associated with the medium, including conductivity current.

(There are no external or "free" currents)

Heating rate is the "systematic influx of energy (per unit time per unit volume) from external sources of radiation."

LL, VIII

A) Non-Magnetic Medium

$$\mathbf{J} = \partial \mathbf{P} / \partial t$$

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}_\omega(\mathbf{r})e^{-i\omega t}]$$

$$\mathbf{P}(\mathbf{r},t) = \mathrm{Re}[\mathbf{P}_\omega(\mathbf{r})e^{-i\omega t}]$$

$$\mathbf{J}(\mathbf{r},t) = \mathrm{Re}[\mathbf{J}_\omega(\mathbf{r})e^{-i\omega t}]$$

$$\mathbf{P}_\omega(\mathbf{r}) = \frac{\varepsilon_\omega - 1}{4\pi}\mathbf{E}_\omega(\mathbf{r}) \;;\;\; \mathbf{J}_\omega(\mathbf{r}) = -i\omega\mathbf{P}_\omega(\mathbf{r})$$

$$q = \frac{1}{2}\mathrm{Re}[\mathbf{J}_\omega^* \cdot \mathbf{E}_\omega] = \frac{\omega}{8\pi}\left|\mathbf{E}_\omega\right|^2 \mathrm{Im}(\varepsilon_\omega)$$

The conventional result

B) Magnetic Medium (The Volume Term)

(i) $\mathbf{r} \in V \ , \quad \nabla \mu_\omega = 0$

$$\mathbf{J} = \partial \mathbf{P} / \partial t + c \nabla \times \mathbf{M}$$

$$\mathbf{J}_\omega = -i\omega \frac{\varepsilon_\omega - 1}{4\pi} \mathbf{E}_\omega + c \nabla \times \frac{\mu_\omega - 1}{4\pi} \mathbf{H}_\omega$$

$$\mathbf{J}_\omega = -i\omega \frac{\varepsilon_\omega - 1}{4\pi} \mathbf{E}_\omega + c \frac{\mu_\omega - 1}{4\pi} \nabla \times \mathbf{H}_\omega$$

$$\nabla \times \mathbf{H}_\omega = -i \frac{\omega}{c} \mathbf{D}_\omega = -i \frac{\omega}{c} \varepsilon_\omega \mathbf{E}_\omega$$

$$\mathbf{J}_\omega = \frac{\omega}{4\pi i} \left[\mu_\omega \varepsilon_\omega - 1 \right] \mathbf{E}_\omega$$

$$q^{(V)} = \frac{\omega \left| \mathbf{E}_\omega \right|^2}{8\pi} \mathrm{Im}\left[\mu_\omega \varepsilon_\omega \right]$$

This is different from the conventional result

B) Magnetic Medium (The Surface Term)

(ii) $\mathbf{r} \in S = \partial V \;,\quad \nabla \mu_\omega(\mathbf{r}) = (\mu_\omega - 1)\hat{\mathbf{n}}_\mathbf{R}\delta\left(\hat{\mathbf{n}}_\mathbf{R} \cdot (\mathbf{r} - \mathbf{R})\right)$

R - point on the surface

$\hat{\mathbf{n}}_\mathbf{R}$ - outward unit normal drawn at point **R**

$$q^{(S)}(\mathbf{R}) = \frac{\omega}{8\pi}\mathrm{Re}\left[(1-\mu_\omega)\left(\mathbf{H}_\omega \times \mathbf{E}_\omega^*\right)\cdot \hat{\mathbf{n}}_\mathbf{R}\right]$$

$$Q = \underbrace{\int q^{(V)}(\mathbf{r})d^3r}_{Q^{(V)}} + \underbrace{\int q^{(S)}(\mathbf{R})d^2R}_{Q^{(S)}}$$

$$= Q^{(V)} + Q^{(S)}$$

(total heat absorbed by the body per unit time)

Intermediate Summary

The conventional result

$$q_{\text{conv}} = \frac{\omega}{8\pi}\left[\left|\mathbf{E}_\omega\right|^2 \varepsilon''_\omega + \left|\mathbf{H}_\omega\right|^2 \mu''_\omega\right]$$

$$Q_{\text{conv}} = \int q_{\text{conv}}(\mathbf{r})\,d^3r \qquad \text{(volume term only)}$$

My result

$$q_{\text{my}}^{(V)} = \frac{\omega\left|\mathbf{E}_\omega\right|^2}{8\pi}\text{Im}\left[\mu_\omega \varepsilon_\omega\right]$$

$$q_{\text{my}}^{(S)}(\mathbf{R}) = \frac{\omega}{8\pi}\text{Re}\left[(1-\mu_\omega)\left(\mathbf{H}_\omega \times \mathbf{E}^*_\omega\right)\cdot \hat{\mathbf{n}}_\mathbf{R}\right]$$

$$Q_{\text{my}} = \int q_{\text{my}}^{(V)}(\mathbf{r})\,d^3r + \int q_{\text{my}}^{(S)}(\mathbf{R})\,d^2R$$

It can be shown that

$$Q_{\text{my}} = Q_{\text{conv}}$$

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but

$$q_{\text{my}} \neq q_{\text{conv}}$$

Why the Two Results are Different? (Poynting Theorem)

$$\frac{c}{4\pi}\nabla\cdot(\mathbf{E}\times\mathbf{H})+\frac{1}{4\pi}\left(\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right)=0$$

This is an identity which follows from Maxwell equations

If we identify $\mathbf{S}=\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$ **(this is simply postulated)**

Then, in the statioary case (i.e., for monochromatic fields),

$\langle\nabla\cdot\mathbf{S}\rangle_{\text{time}}+q=0$ ... and

$$q=q_{\text{conv}}=\frac{1}{4\pi}\left\langle\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right\rangle_{\text{time}}$$

The Poynting Theorem (Cont.)

But $\left\langle \frac{1}{4\pi}\left(\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right)\right\rangle \neq \left\langle \mathbf{J}\cdot\mathbf{E}\right\rangle$!!!

(the equality holds in non-magnetic media only).

⇓

There is no physical basis to identify $\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$ as the current of energy (which, by definition, is the Poynting vector).

The formula $\mathbf{S}=\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$ is incorrect

Are there any other problems with the expression $\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$?

* **H** is not a physical field...

* What if the responce is nonlinear? Moreover, what if there is no one-to-one correspondence between **M** (or **B**) and **H** as in the case for ferromagnetics?

* You can always write

$$\nabla\cdot\left(\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}+\frac{1}{4\pi}\mathbf{F}\right)+\frac{1}{4\pi}\left(\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}-\nabla\cdot\mathbf{F}\right)=0$$

... where **F** is arbitrary (not necessarily solenoidal!)

The Poynting Theorem (a side note)

I view here the well-known invariance of the Poynting's identity with respect to the transformation

$$\mathbf{S} \rightarrow \mathbf{S} + \nabla \times \mathbf{f}$$

(which indeed has received much attention in the past) as a non-issue.

I believe, only $\nabla \cdot \mathbf{S}$ is observable and has, therefore, any physical significance.

What's the Right Expression for **S** ?

Then what is the correct expression for the Poynting vector?

a) A hint (less fundamental)

$$\mathbf{S} = \frac{c}{4\pi}\langle \mathbf{e}\times\mathbf{h}\rangle_{\text{volume}} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} + \text{terms quadratic in fluctuations}$$

b) From first principles (more fundamental)

If we take $\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B}$, then

$$\langle \nabla\cdot\mathbf{S}\rangle_{\text{time}} + \langle \mathbf{J}\cdot\mathbf{E}\rangle_{\text{time}} = 0$$

and $$q = \langle \mathbf{J}\cdot\mathbf{E}\rangle_{\text{time}} = q_{\text{my}}$$

Heating Rate and Negative Refraction

$$q_{\mathrm{my}}^{(V)} = \frac{\omega \left|\mathbf{E}_\omega\right|^2}{8\pi} \mathrm{Im}\left[\mu_\omega \varepsilon_\omega\right]$$

$$q_{\mathrm{my}}^{(S)} = \frac{\omega}{8\pi} \mathrm{Re}\left[\left(1-\mu_\omega\right)\left(\mathbf{H}_\omega \times \mathbf{E}_\omega^*\right)\cdot \hat{\mathbf{n}}_{\mathbf{R}}\right]$$

$$Q_{\mathrm{my}} = Q_{\mathrm{my}}^{(V)} + Q_{\mathrm{my}}^{(S)} = Q_{\mathrm{conv}} > 0$$

In NR materials, if such existed, external radiation would extract thermal energy from the volume (cool the volume) and deposit it on the surface....

Is this possible?

What Does the Second Law Say?

Clausius: There exists no thermodynamic transformation whose ***sole effect*** is to extract a quantity of heat from a colder reservoir and to deliver it to a hotter reservoir

(Heat does not flow spontaneously from a cold object to a hot object)

Kelvin: There exists no thermodynamic transformation whose ***sole effect*** is to extract a quantity of heat from reservoir and to transform it entirely into work

(Perpetual motion machine of the second kind – the "heat engine" – is impossible)

BUT IN "METAMATERIALS" THE VOLUME CERTAINLY CAN NOT BE COOLED. THEREFORE, NR IS IMPOSSIBLE IN SUCH MATERIALS

Anisotropic Media: A Brief Summary

If the medium can support plane waves

of the type $\mathbf{E} = \mathrm{Re}[\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}]$ as solutions to the Maxwell's equations,

then $q_{\mathrm{my}}^{(V)} = \dfrac{\omega e^{-2\mathbf{k}''\cdot\mathbf{r}}}{8\pi(\omega/c)^2} \mathrm{Im}\left[|\mathbf{E}_0|^2 (\mathbf{k}\cdot\mathbf{k}) - (\mathbf{k}\cdot\mathbf{E}_0)(\mathbf{k}\cdot\mathbf{E}_0^*) \right]$

A wave is "propagating" if

$\mathbf{k} = \hat{\mathbf{u}} k$

($\hat{\mathbf{u}}$ - purely real unit vector; k - complex scalar).

Otherwise, the wave is evanecent.

For a propagating wave, $(\mathbf{k}\cdot\mathbf{E}_0)(\mathbf{k}\cdot\mathbf{E}_0^*) = (\mathbf{k}\cdot\mathbf{k})|\mathbf{E}_0|^2 \cos^2\theta$

and

$q_{\mathrm{my}}^{(V)} \propto \sin^2\theta \, \mathrm{Im}(\mathbf{k}\cdot\mathbf{k})$

Objections

1. “You have assumed that only electric field does work on moving charges. But magnetic field can also do work, e.g., on magnetic moments. Otherwise, how do you explain ….” [choose your example]

Dead men tell no tales and magnetic fields do no work

Objections (Cont.)

2. "May be, effective medium parameters describe (approximately) some phenomena associated with wave propagation through composite media but not *all* such phenomena."

May be. But then, macroscopic Maxwell's equations can't be used without restriction.

Also, experimentally measurable quantities, such as the intensity, are bilinear in the fields. Therefore, any useful homogenization model must correctly predict such quadratic combinations, including the Poynting vector and the heating rate.

Objections (Cont.)

3. "The electric and magnetic currents are different in physical origin and the same laws of motion can not be applied to them."

There is neither a physical nor a mathematical way to disentangle the terms $\partial \mathbf{P} / \partial t$ and $c\nabla \times \mathbf{M}$ from the expression for the total current

$$\mathbf{J} = \partial \mathbf{P} / \partial t + c\nabla \times \mathbf{M}$$

All laws or formulas must be applied to the total current $\mathbf{J}$.

Objections (Cont.)

4. “What about the zero-frequency limit? Won’t you obtain unphysical results at the surface?”

First, we are not really talking about low frequency magnetism.

But there is no unphysical effects in my theory in the zero-frequency limit. In fact, a careful analysis reveals that the conventional theory predicts an electromagnetic equilibrium for a magnetized object in external static E-field which contradicts mechanical equilibrium of charges. My theory is free of such contradictions.

Objections (Cont.)

5. “What about all those experiments ?”

Show me your superselns.

(and it’s been in the works for about ten years)

More Seriously: An Experimental Example…

"Comparison of the measured and calculated angles for the NIM in Fig.3 shows good agreement for at least ±10% about the normal incidence and modest deviation at higher angles…"

How modest is modest and how normal is normal?
Anyway, 10% precision seems awfully inadequate for something as fundamental as Maxwell's eqs.

"Although the measured angles of the refracted signal closely followed the prediction, the measured amplitudes did not…. Such increased attenuation [inconsistent with the theoretical model – V.M.] has implications for off-axis incidence … and is under further investigation."

So how did the further investigation go?

Wouldn't it be more logical to conclude that the medium CAN NOT be characterized by effective parameters?

Quoted from: J.S.Derov, B.W.Turchinetz, E.E.Crisman, A.J.Drehman, S.R.Best, and R.M.Wing, IEEE Microwave and Wireless Components Letters 15, 567 (2005)

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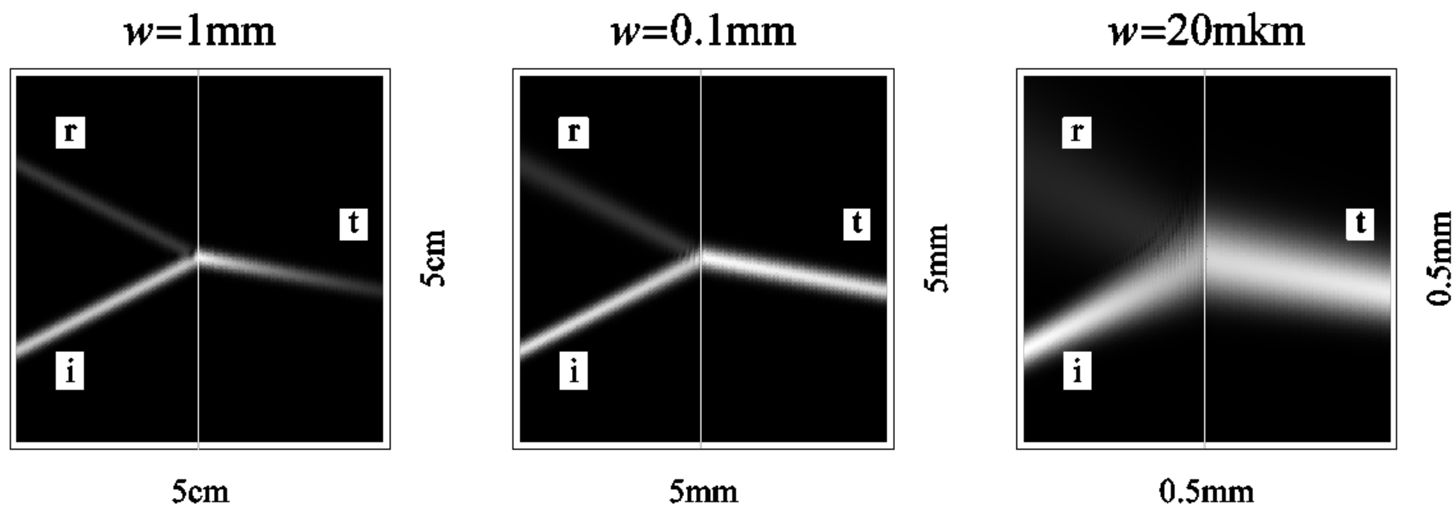
Refraction of a p-polarized Gaussian beam at an interface with a highly anisotropic (layered) medium



Numerical simulations for the layered medium fabricated in
[A.J.Hoffman et al., Nature Photonics 6, 946 (2007)]
…. but with significantly reduced losses

$$\varepsilon_{xx} = \varepsilon_{yy} = 4.12 \ , \ \ \varepsilon_{zz} = -4.92$$

(The refracted beam is a superposition of positively-refracted plane waves)

This is Why

- **THERE IS NO "EXPONENTIAL AMPLIFICATION" OF EVANESCENT WAVES, NO MATTER HOW SMALL THE ABSORPTION IS**
- **THE REFRACTED BEAMS I'VE SHOWN ARE SUPERPOSITIONS OF POSITIVELY REFRACTED PLANE WAVES. THE OPTICAL PHASE OF THE BEAM INCREASES AWAY FROM THE INTERFACE**
- **THERE ARE MANY DIFFERENT MEANS TO DEFLECT A BEAM IN ANY GIVEN DIRECTION: DIFFRACTION GRATING, ANISOTROPY, A MIRROR…**

  BUT SO WHAT?

See A.D.Boardman et al., "Negative refraction in perspective," Electromagnetics 25, 365 (2005)

"Statements surrounding the possible appearance of NR can be exaggerated, however, sometimes misleading, and at best limited in scope. For example, anisotropic crystals such as calcite can exhibit BUT amphoteric [positive or negative – V.M.] refraction ….. The linkage of this to the general bandwagon associated with negative phae velocity media is incorrect…"

(pp.381-382 of the above reference)

But can there be true NR in anisotropic crystals?

According to my paper, the answer is YES, but only for evanescent waves (and still there will be no "exponential amplification")

For example, in a melal-dielectric layered medium (well below the plasma frequency), NR is possible only if

$$k_x^2 > k_{cx}^2 \approx k_0^2 \frac{\varepsilon_d}{1-p_m} \ , \quad k_{tz}^2 < k_{cz}^2 \approx -2k_0^2 p_m |\varepsilon_m|$$

$$k_0 = \frac{\omega}{c} \ , \ p_m \text{ - volume fraction of metal}$$

The wave is evanescent both in vacuum and in the medium