SINGLE-SCATTERING OPTICAL TOMOGRAPHY

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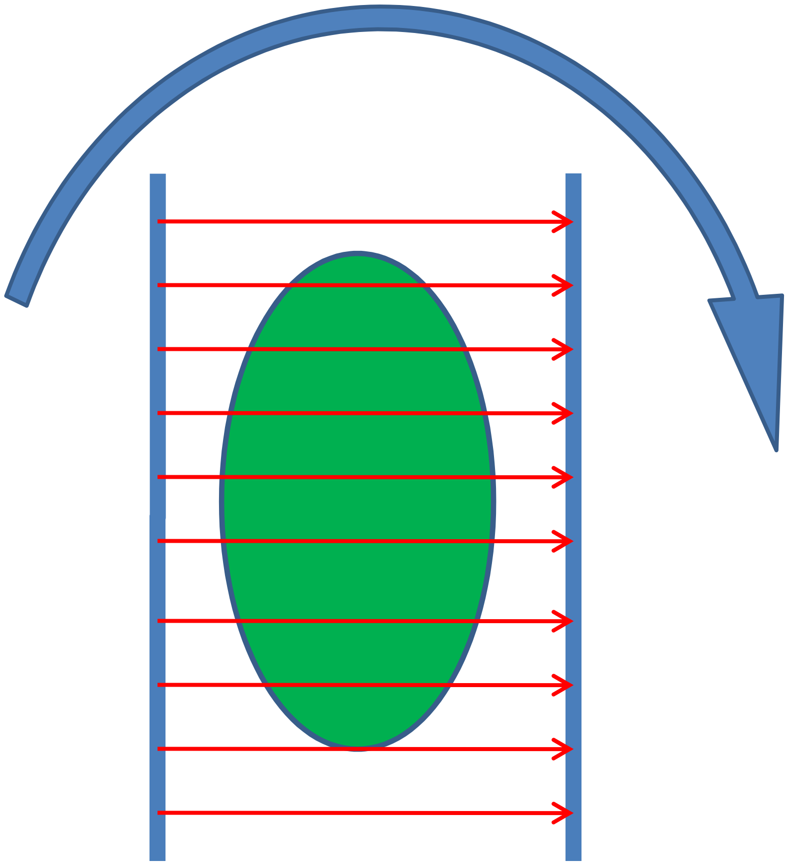
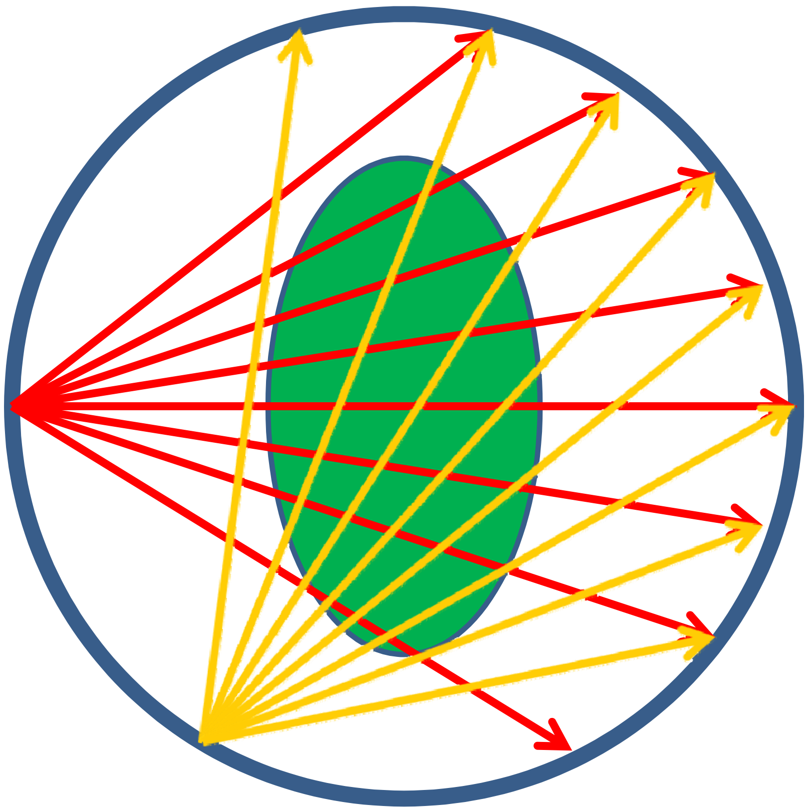
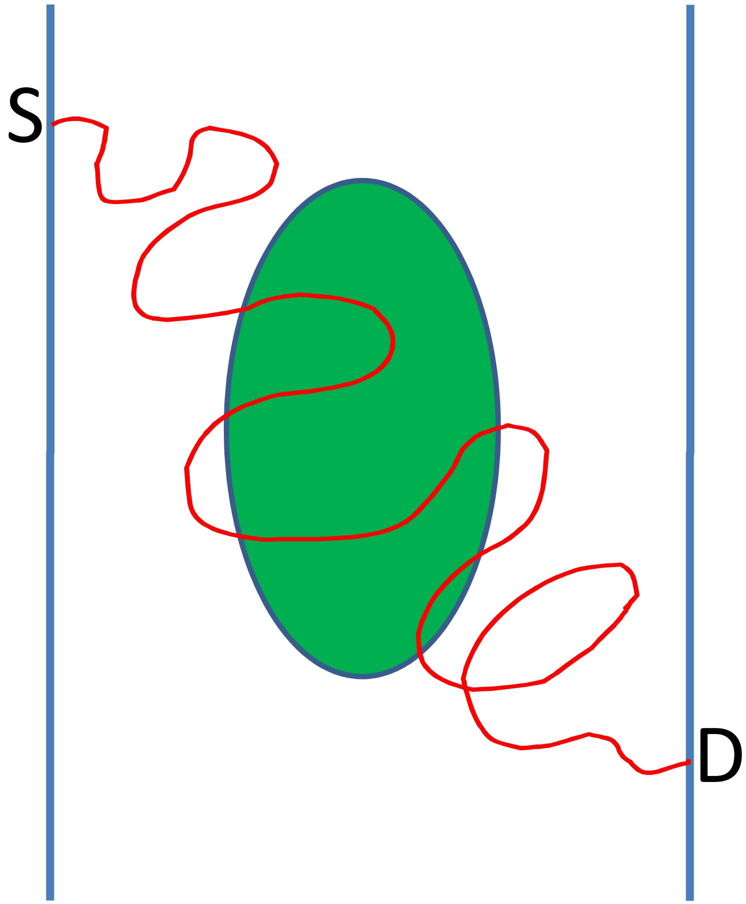
Zero Scattering Regime: Conventional X-Ray Tomography



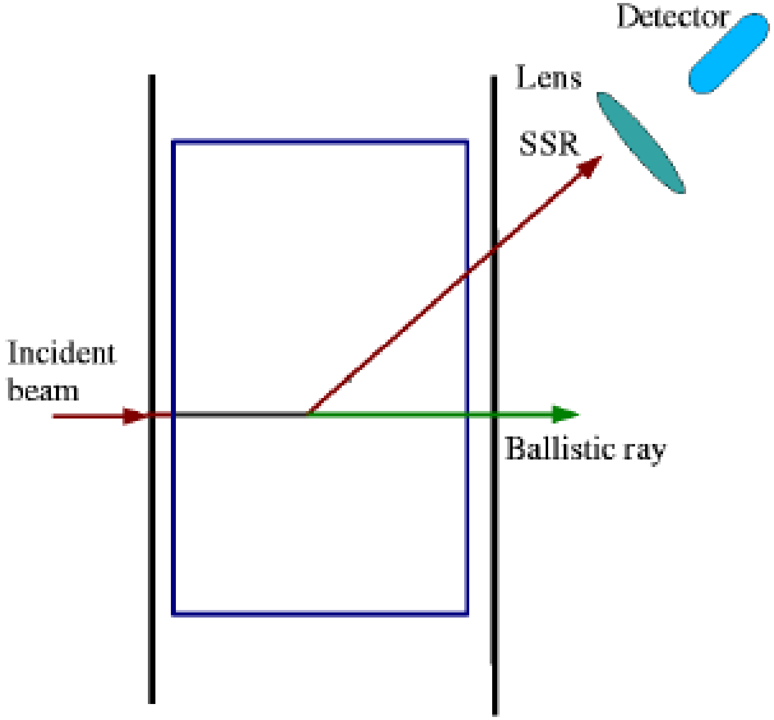


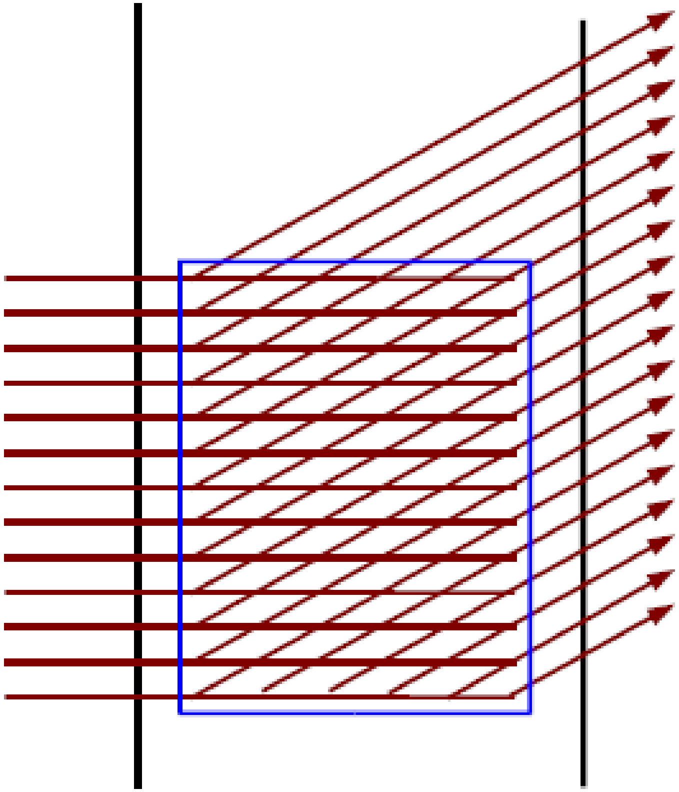
Strong Scattering Regime: Diffuse Optical Tomography



- Many source-detector pairs
- Severely ill-posed IP
- Nonlinear IP

Mesoscopic Scattering Regime: Single-Scattering Tomography





SSOT And Other Modalities

	Linear IP	IP is MILDLY Ill-posed	Single projection	Reflection geometry	Nonionizing radiation	Quantitatrive images	New contrast mechanisms
X-ray CT	Y (if *E*=const)	Y	N	N	N	Y	N
DOT	N	N	Y	Y	Y	?	Y
SSOT	Y	Y	Y	Y	Y	Y	Y

The Broken-Ray Integral Transform

(a) $\mu_s = \text{const}$ (and is known)

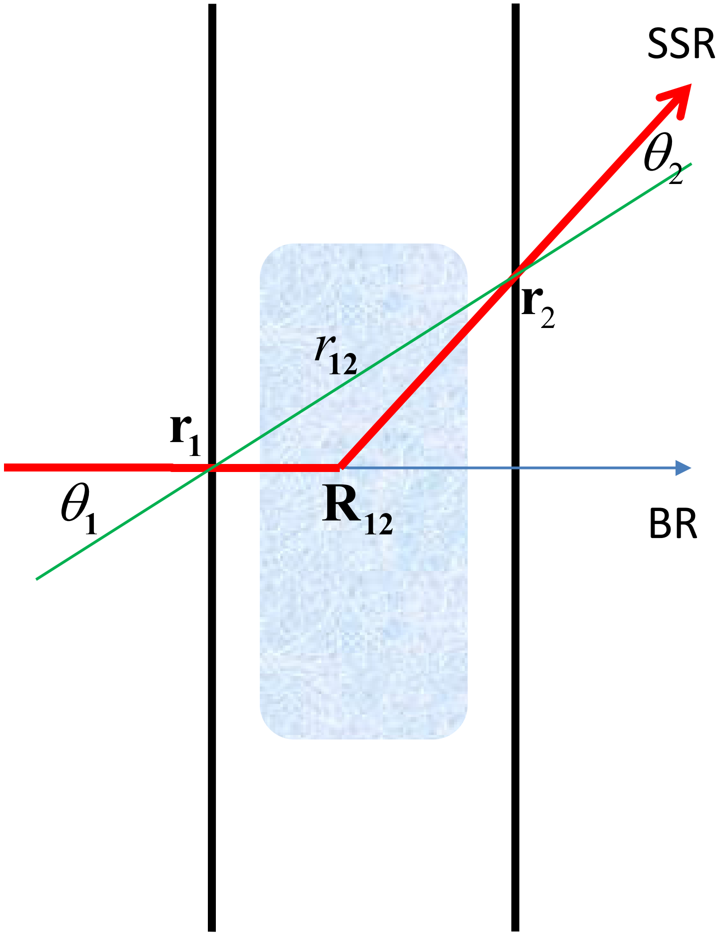
$$\int_{\mathrm{SSR}(\mathbf{r}_1,\hat{\mathbf{s}}_1;\mathbf{r}_2,\hat{\mathbf{s}}_2)} \mu_t[\mathbf{r}(\ell)]d\ell = \phi(\mathbf{r}_1,\hat{\mathbf{s}}_1;\mathbf{r}_2,\hat{\mathbf{s}}_2)$$

(b) $\mu_s \neq \text{const}$ (and is unknown)

$$\int_{\mathrm{SSR}(\mathbf{r}_1,\hat{\mathbf{s}}_1;\mathbf{r}_2,\hat{\mathbf{s}}_2)} \mu_t[\mathbf{r}(\ell)]d\ell - \ln\left[\frac{\mu_s(\mathbf{R}_{12})}{\langle\mu_s\rangle}\right] = \phi(\mathbf{r}_1,\hat{\mathbf{s}}_1;\mathbf{r}_2,\hat{\mathbf{s}}_2)$$

The measurable data function:

$$\phi(\mathbf{r}_1,\hat{\mathbf{s}}_1;\mathbf{r}_2,\hat{\mathbf{s}}_2) = -\ln\left[\frac{r_{12}\sin\theta_1\sin\theta_2}{\langle\mu_s\rangle A(\hat{\mathbf{s}}_1,\hat{\mathbf{s}}_2)}\frac{I_{\text{measured}}}{I_{\text{incident}}}\right]$$



SIMULATIONS

- Forward model based on the RTE
- Isotropic scattering
- FULL ACCOUNT OF MULTIPLE SCATTERING
- BOUNDARY CONDITIONS SATISFIED EXACTLY
- 3D integral equation for density discretized on a rectangular grid
- Direct inversion of a well-posed square matrix
- Mathematical details on next page…

$$[\hat{\mathbf{s}}\cdot\nabla + \underbrace{\mu_a(\mathbf{r}) + \mu_s(\mathbf{r})}_{\mu_t(\mathbf{r})}]I(\mathbf{r},\hat{\mathbf{s}}) = \mu_s(\mathbf{r})\int \underbrace{\frac{1}{4\pi}}_{A(\hat{\mathbf{s}},\hat{\mathbf{s}}') = \frac{1}{4\pi} = \text{const}} I(\mathbf{r},\hat{\mathbf{s}}')d^2\hat{\mathbf{s}}'$$

$$u(\mathbf{r}) = \int I(\mathbf{r},\hat{\mathbf{s}})d^2\hat{\mathbf{s}}$$

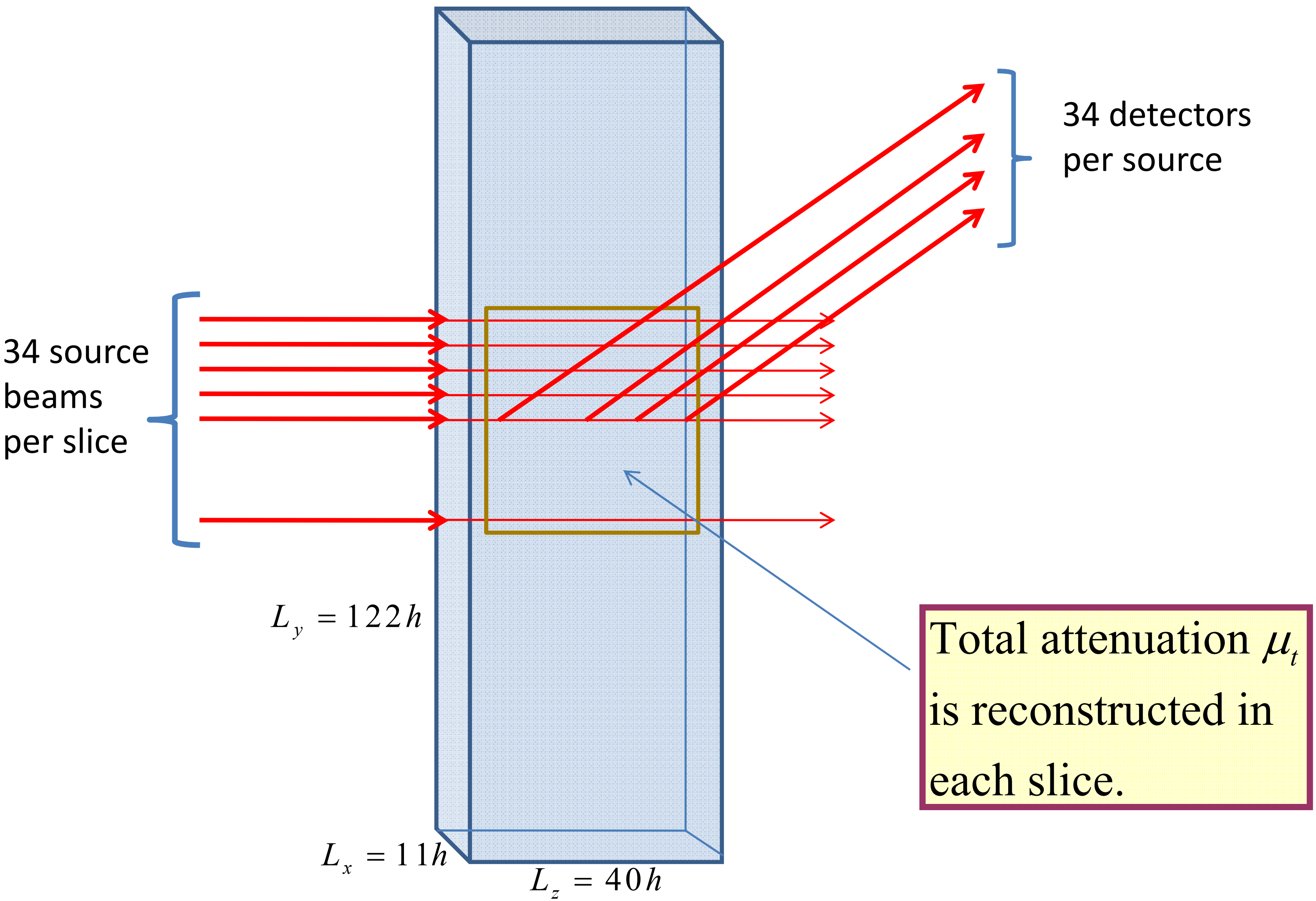
$$u(\mathbf{r}) = u_b(\mathbf{r}) + \int g_b(\mathbf{r},\mathbf{r}')\frac{\mu_s(\mathbf{r}')}{4\pi}u(\mathbf{r}')d^3r'$$

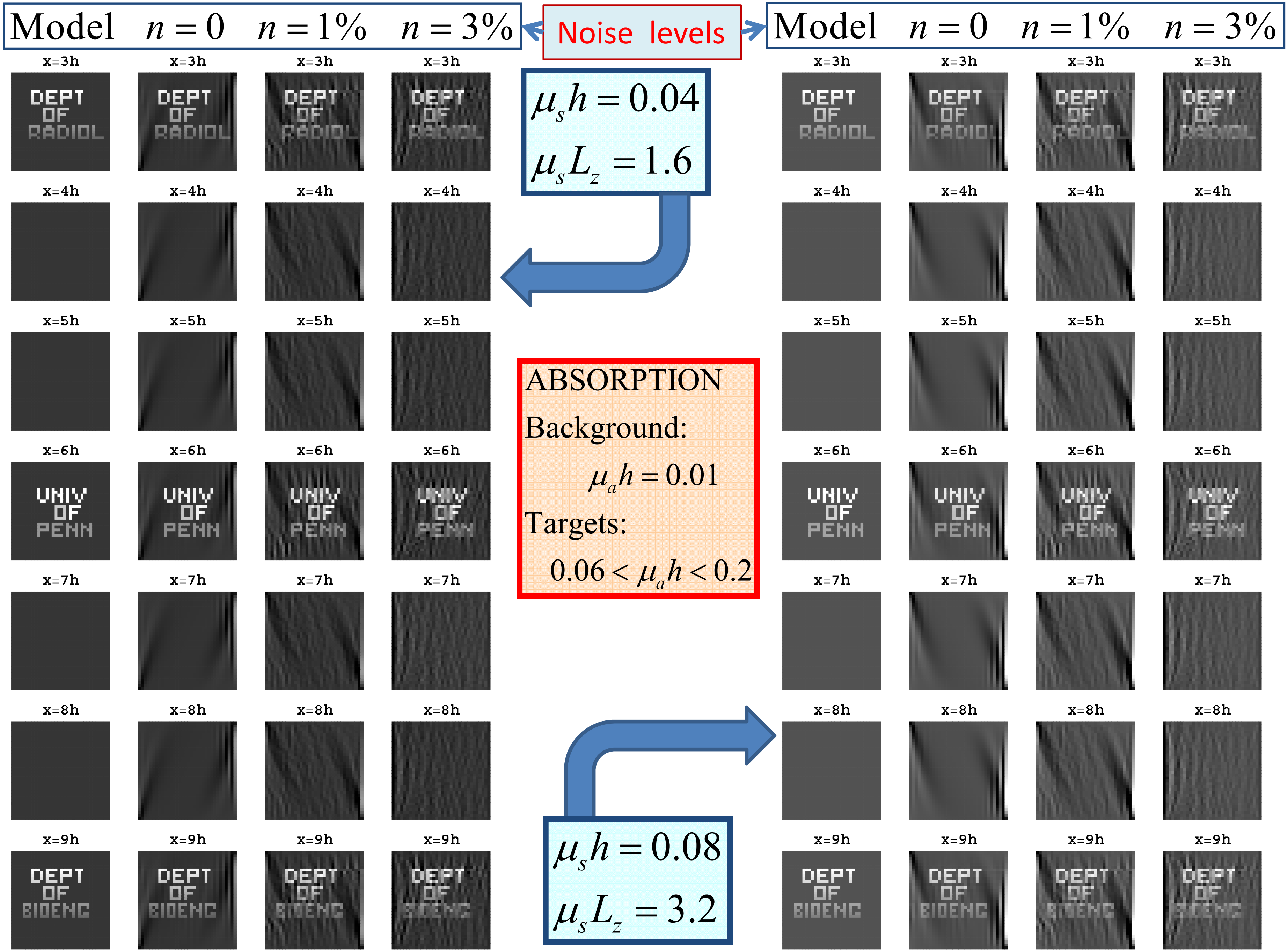
$$I(\mathbf{r},\hat{\mathbf{s}}) = I_b(\mathbf{r},\hat{\mathbf{s}}) + \int G_b(\mathbf{r},\hat{\mathbf{s}};\mathbf{r}',\hat{\mathbf{s}}')\frac{\mu_s(\mathbf{r}')}{4\pi}u(\mathbf{r}')d^3r'd^2\hat{\mathbf{s}}'$$

$$G_b(\mathbf{r},\hat{\mathbf{s}};\mathbf{r}',\hat{\mathbf{s}}') = g_b(\mathbf{r},\mathbf{r}')\delta\left(\hat{\mathbf{u}}(\mathbf{r}-\mathbf{r}')-\hat{\mathbf{s}}'\right)\delta(\hat{\mathbf{s}}-\hat{\mathbf{s}}')$$

$$g_b(\mathbf{r},\mathbf{r}') = \int G_b(\mathbf{r},\hat{\mathbf{s}};\mathbf{r}',\hat{\mathbf{s}}')d^2\hat{\mathbf{s}}d^2\hat{\mathbf{s}}' =$$

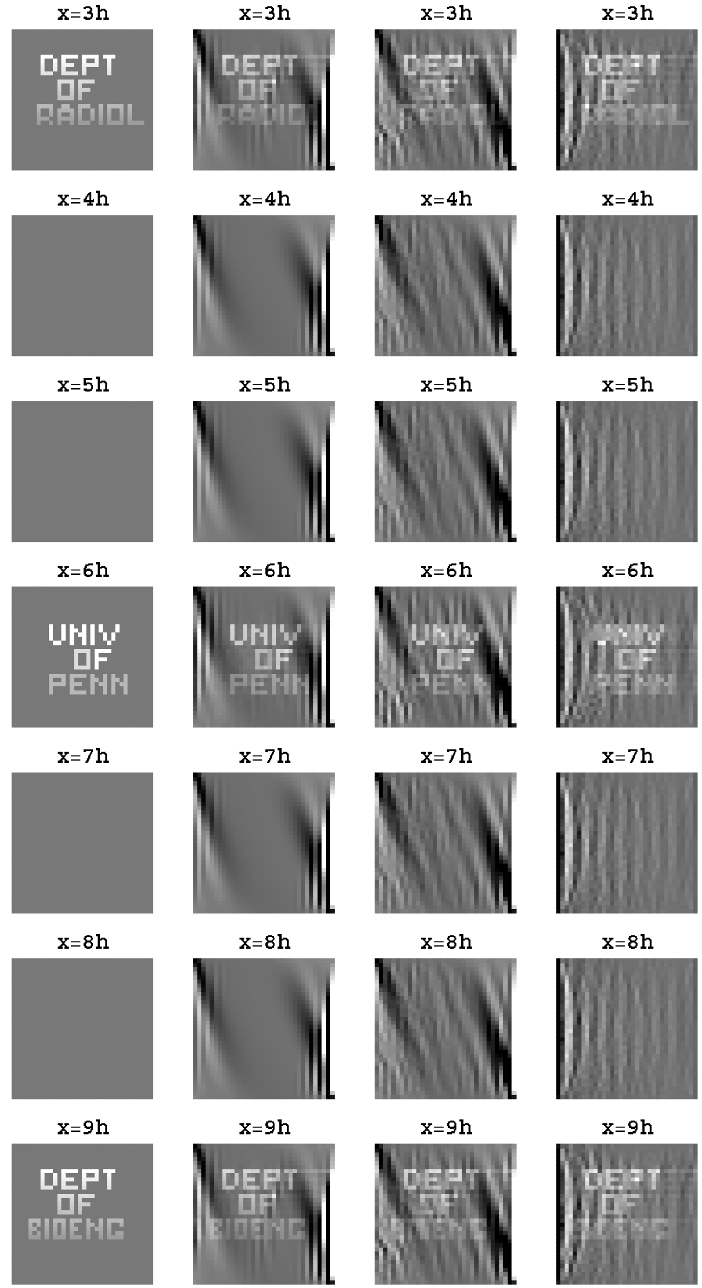
$$= \frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\exp\left[-\int_0^{|\mathbf{r}-\mathbf{r}'|}\mu_t\left(\mathbf{r}'+\ell\hat{\mathbf{u}}(\mathbf{r}-\mathbf{r}')\right)d\ell\right]$$


34 detectors
per source
34 source
beams
per slice
$L_y = 122h$
$L_x = 11h$
$L_z = 40h$
Total attenuation μ_t
is reconstructed in
each slice.



Noise levels
Model $n=0$ $n=1\%$ $n=3\%$
Model $n=0$ $n=1\%$ $n=3\%$
x=3h
x=4h
x=5h
x=6h
x=7h
x=8h
x=9h
DEPT OF RADIOL
UNIV OF PENN
DEPT OF BIOENG
$\mu_s h = 0.04$
$\mu_s L_z = 1.6$
ABSORPTION
Background:
$\mu_a h = 0.01$
Targets:
$0.06 < \mu_a h < 0.2$
$\mu_s h = 0.08$
$\mu_s L_z = 3.2$

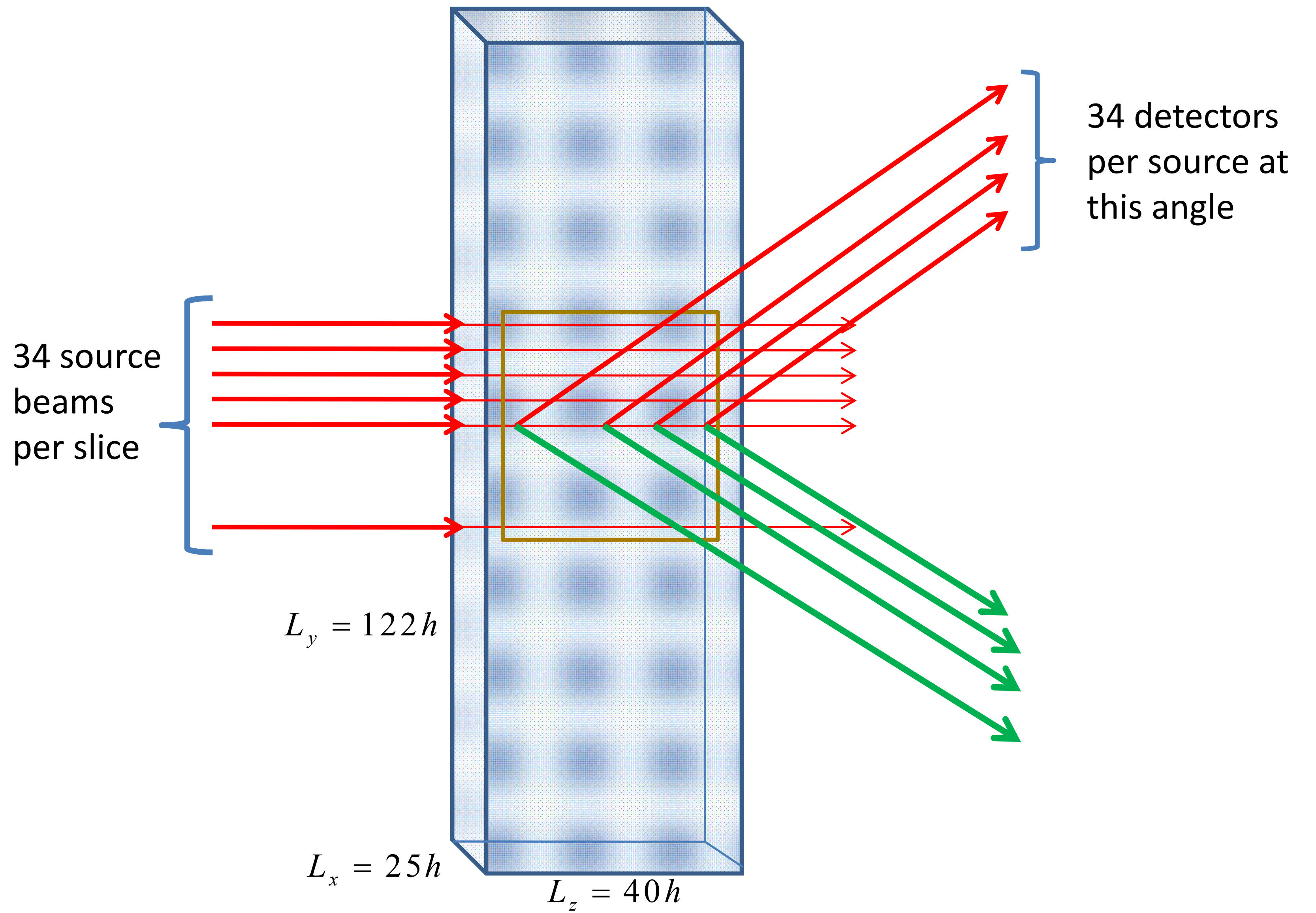
Model $n=0$ $n=1\%$ $n=3\%$



$$\mu_s = 0.16h^{-1}$$

$$\mu_s L_z = 6.4$$

Reconstruction of scattering+absorption in a thicker sample: Lx=25h



Simultaneous reconstruction of absorption and scattering

SCATTERING

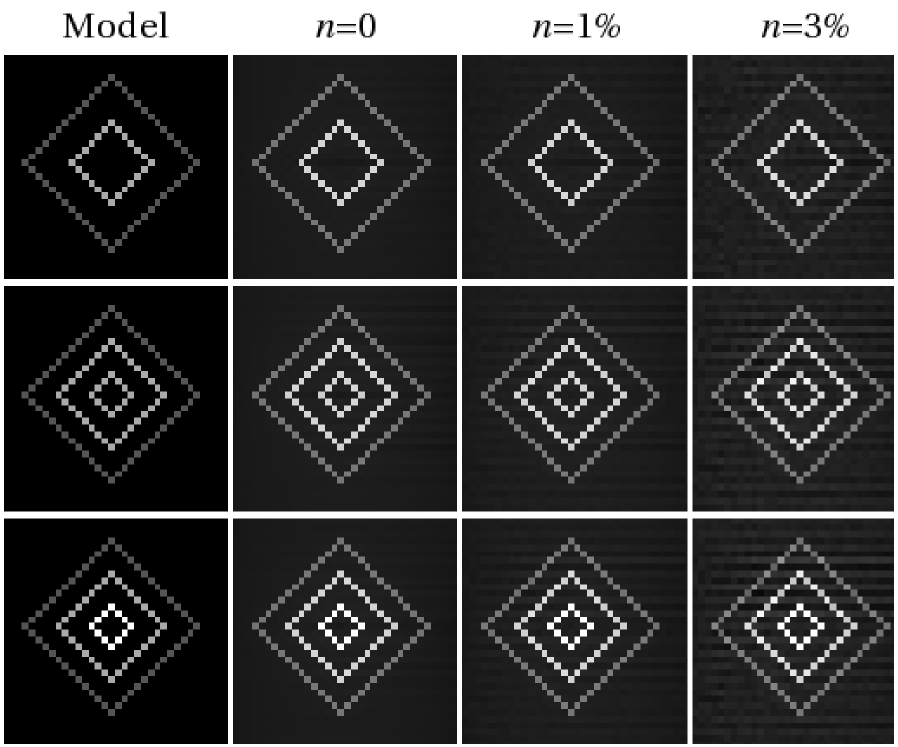
Background: $\overline{\mu}_s L_z = 1.6$

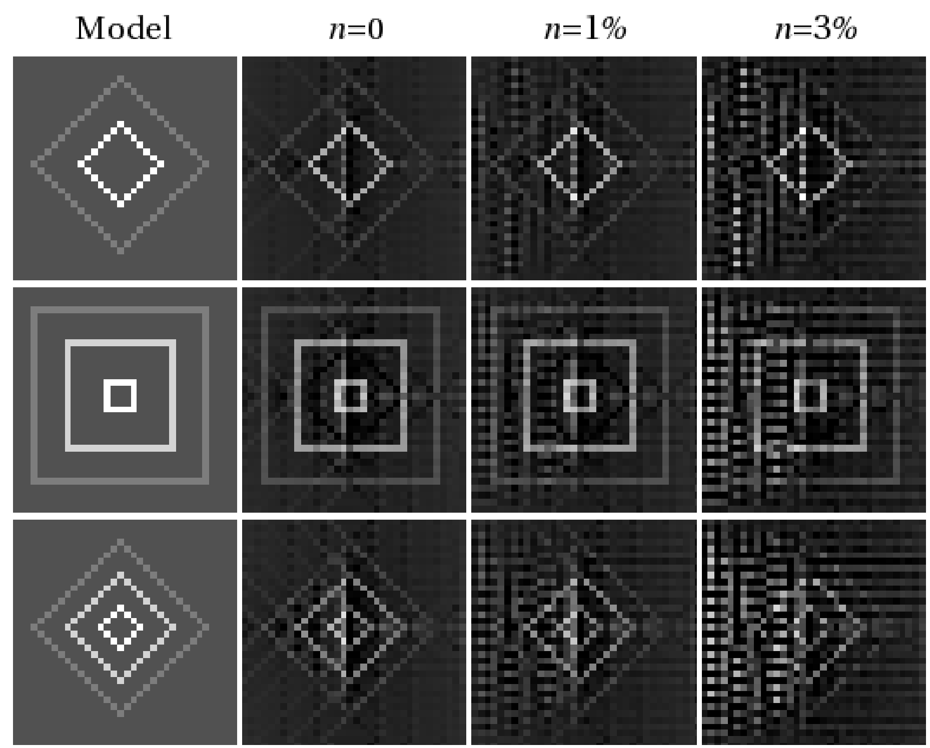
Targets: $1.33\overline{\mu}_s \le \mu_s \le 2\overline{\mu}_s$

ABSORPTION

Background: $\overline{\mu}_a = 0.1\overline{\mu}_s$

Targets : $2\overline{\mu}_a \le \mu_a \le 5\overline{\mu}_a$





Stronger scattering inhomogeneities

SCATTERING

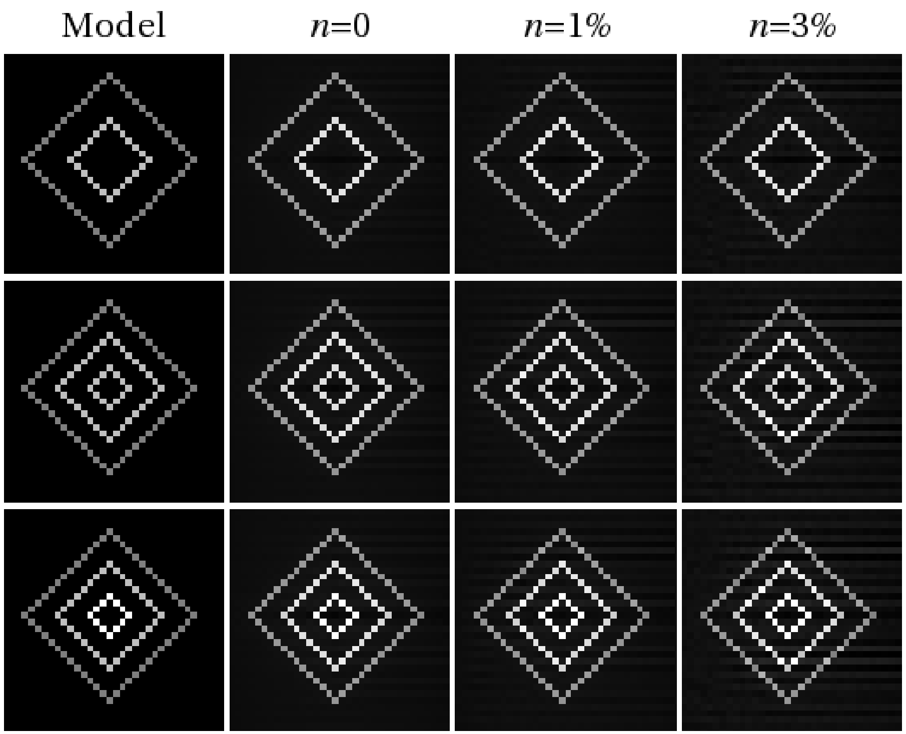
Background: $\overline{\mu}_s L_z = 1.6$

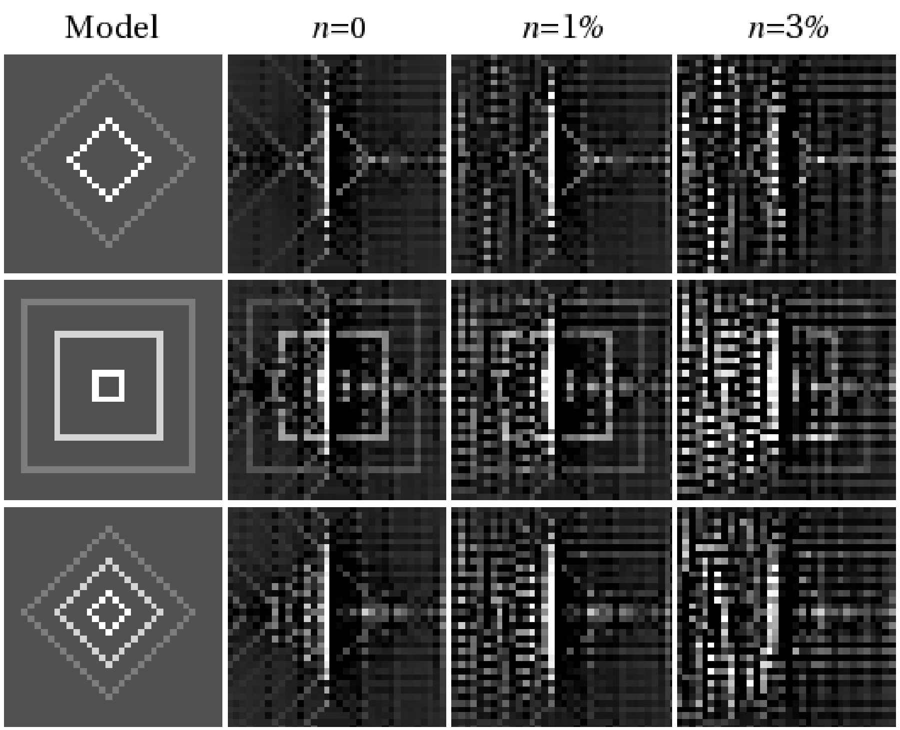
Targets: $2\overline{\mu}_s \le \mu_s \le 3\overline{\mu}_s$

ABSORPTION

Background: $\overline{\mu}_a = 0.1\overline{\mu}_s$

Targets: $2\overline{\mu}_a \le \mu_a \le 5\overline{\mu}_a$





Stronger absorption (overall)

SCATTERING

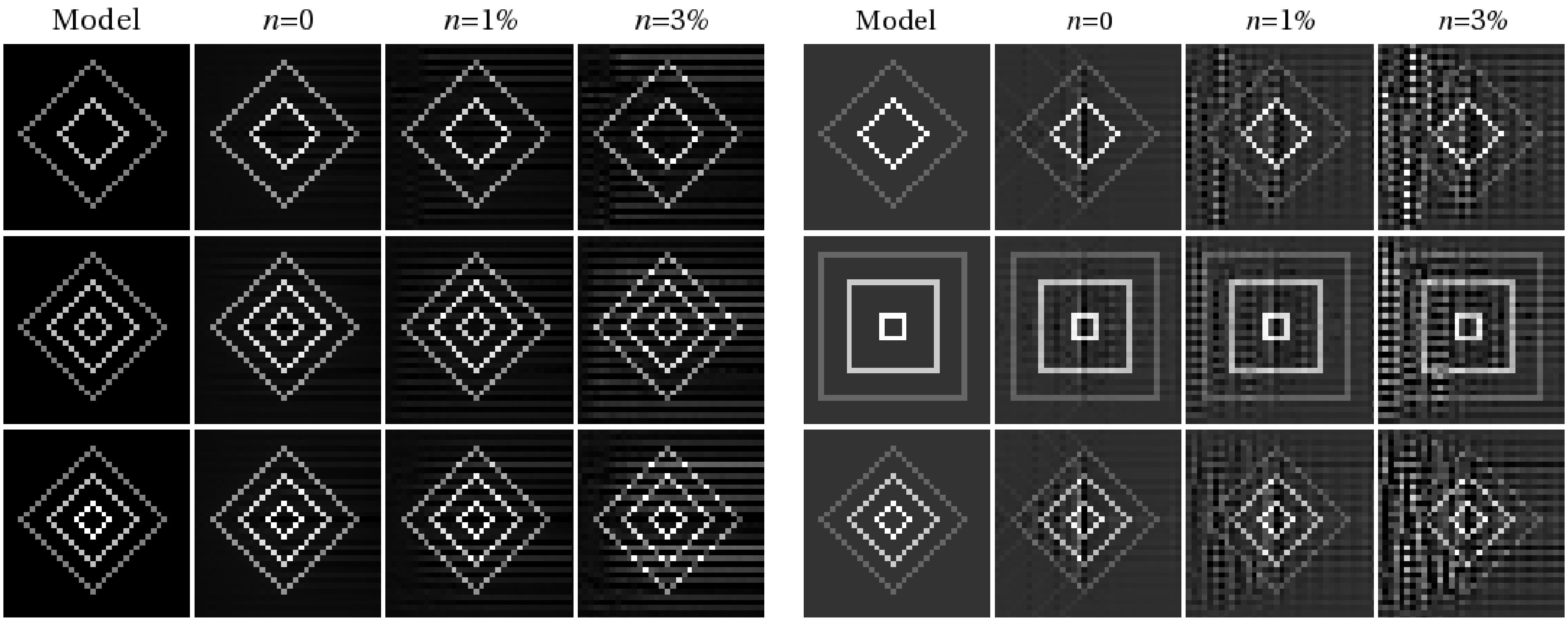
Background: $\overline{\mu}_s L_z = 1.6$

Targets: $2\overline{\mu}_s \leq \mu_s \leq 3\overline{\mu}_s$

ABSORPTION

Background: $\overline{\mu}_a = \overline{\mu}_s$

Targets : $2\overline{\mu}_a \leq \mu_a \leq 5\overline{\mu}_a$



Same as before, but larger optical depth

SCATTERING

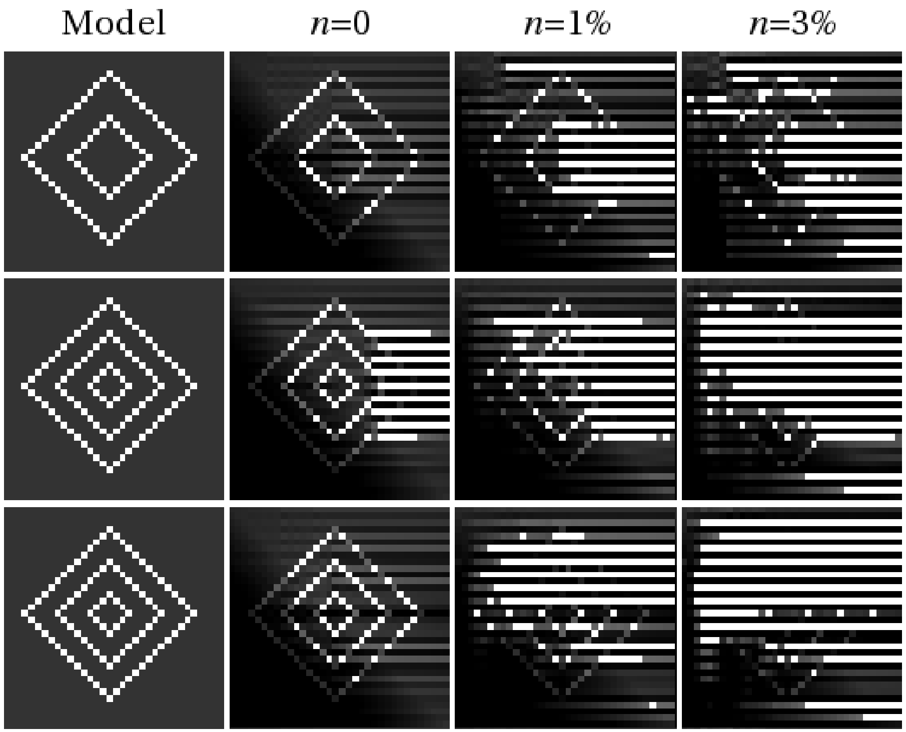
Background: $\overline{\mu}_s L_z = 3.2$

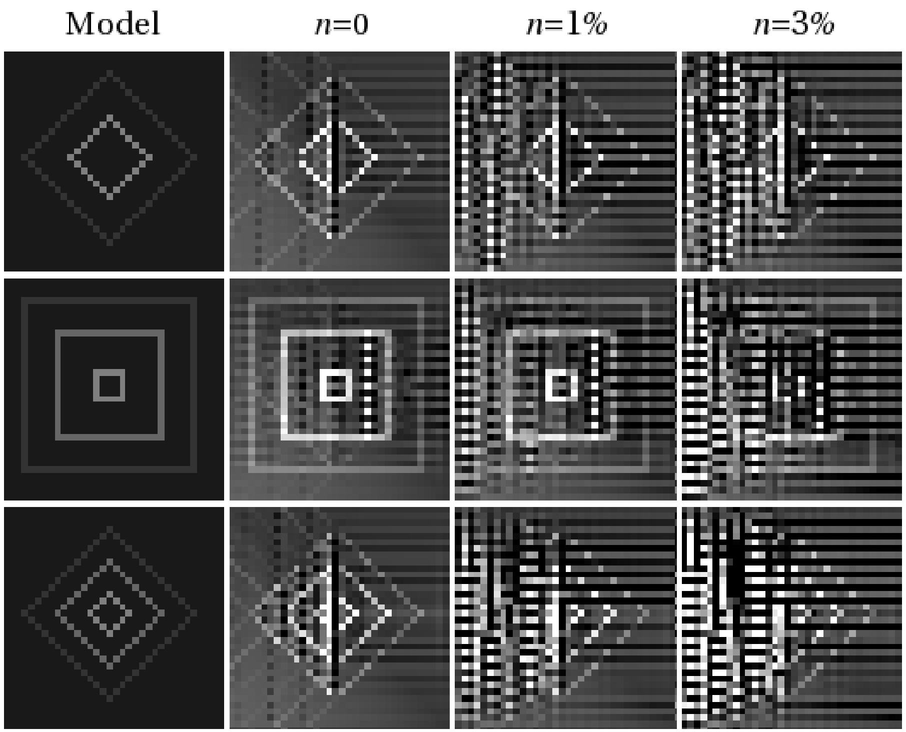
Targets: $2\overline{\mu}_s \le \mu_s \le 3\overline{\mu}_s$

ABSORPTION

Background: $\overline{\mu}_a = \overline{\mu}_s$

Targets: $2\overline{\mu}_a \le \mu_a \le 5\overline{\mu}_a$

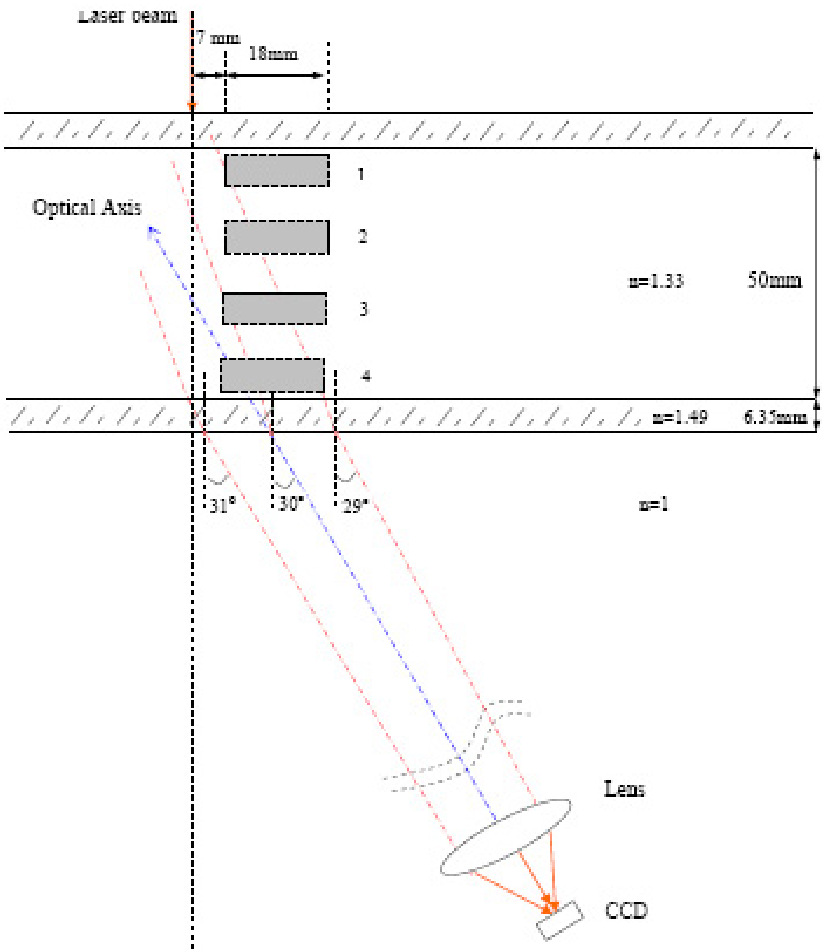


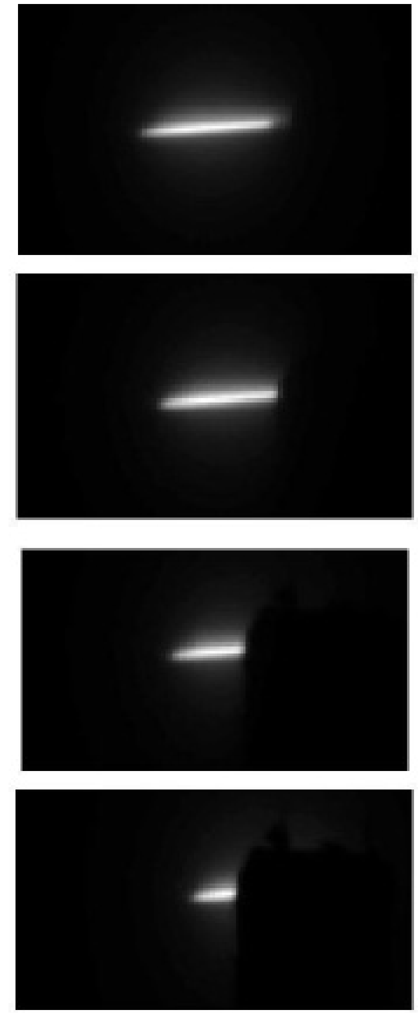


SUMMARY

- SSOT allows accurate quantitative reconstruction of the attenuation function.
- With additional measurements, scattering and absorption can be reconstructed separately
- Ill-posedness of the inverse problem is very mild.
- Tomographic imaging is feasible up to about six scattering lengths, with the noise-to-signal level of about 3% or less.

Preliminary Experiment



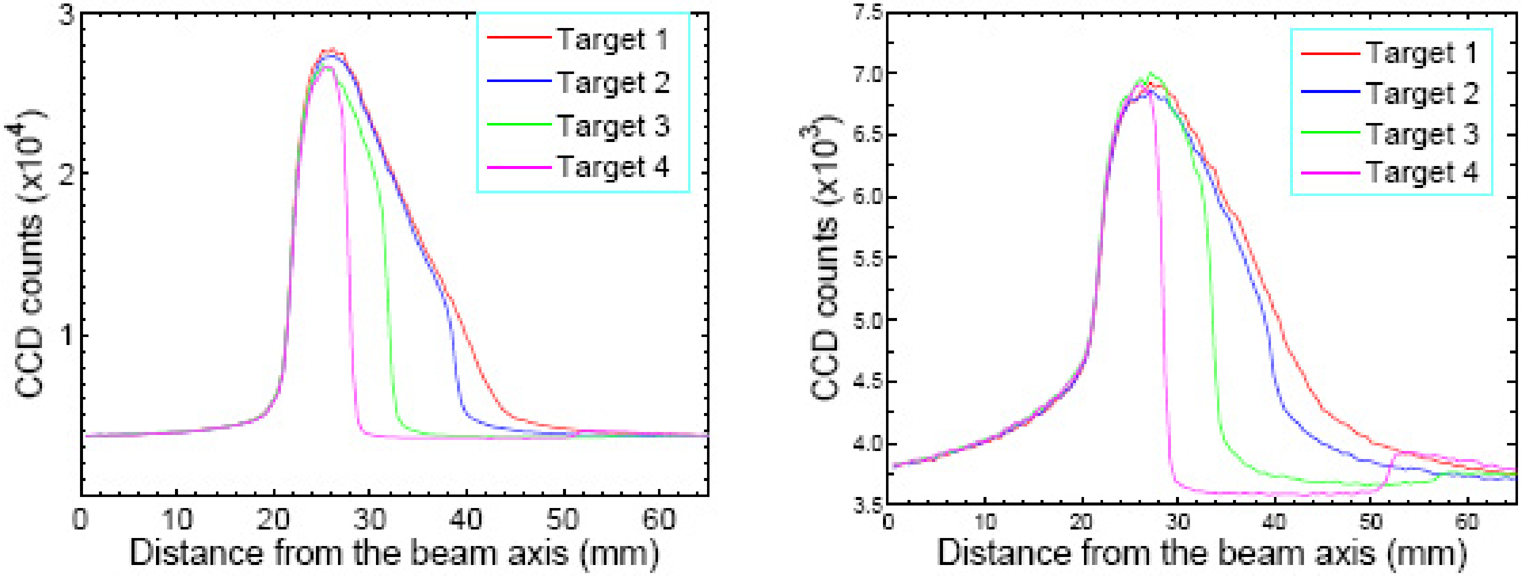


1

2

3

4



(a) (b)

Figure 6: Experimental measurements of the specific intensity as a function of the exit position on the slab surface for intralipid concentrations 0.02% (a) and 0.04% (b).