

SINGLE-SCATTERING OPTICAL TOMOGRAPHY

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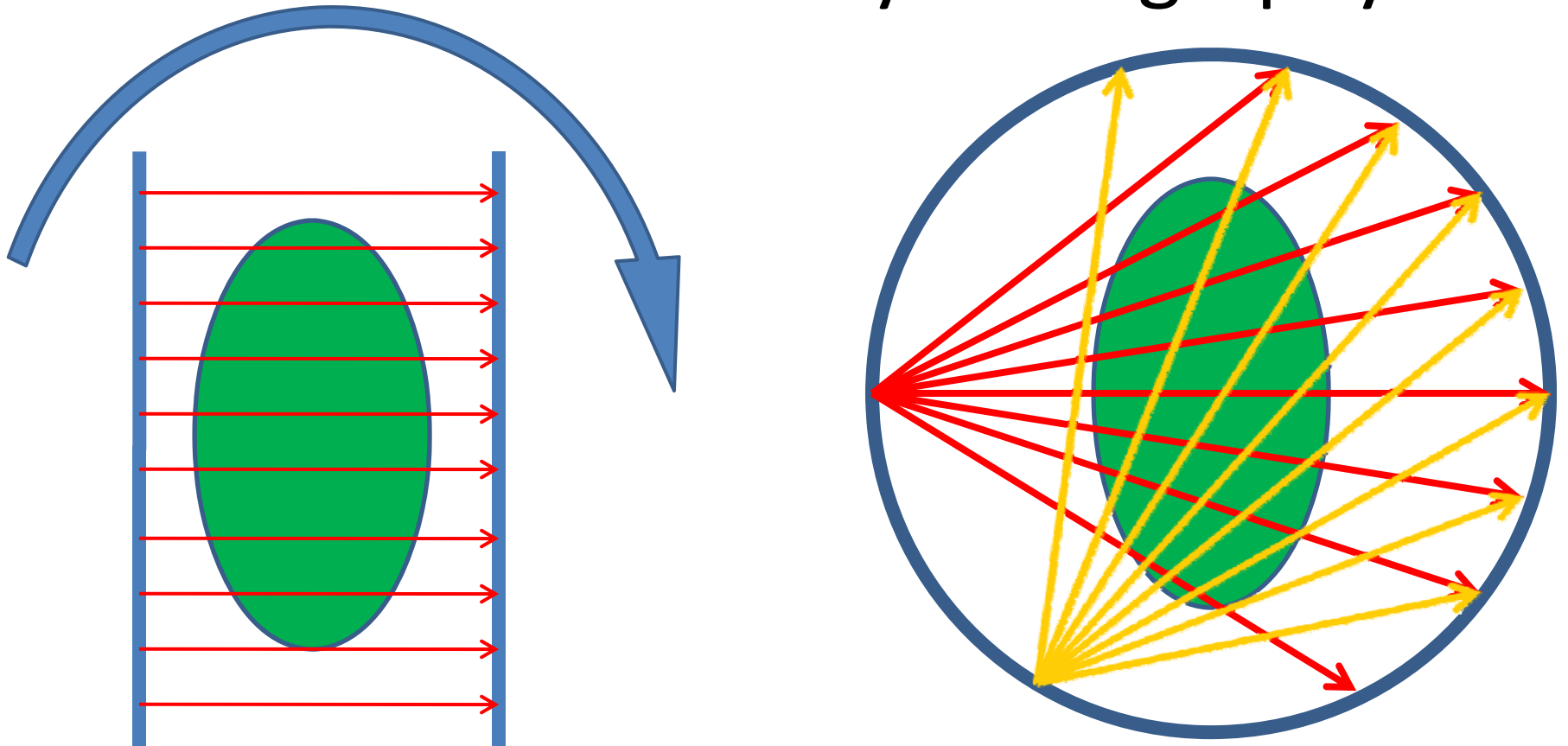
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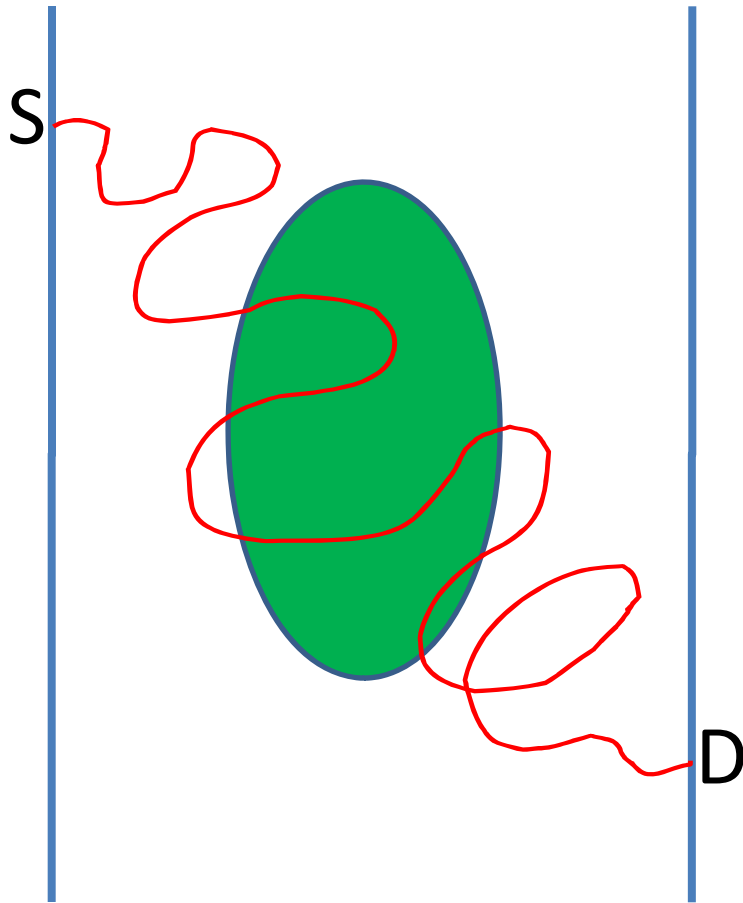
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Zero Scattering Regime: Conventional X-Ray Tomography

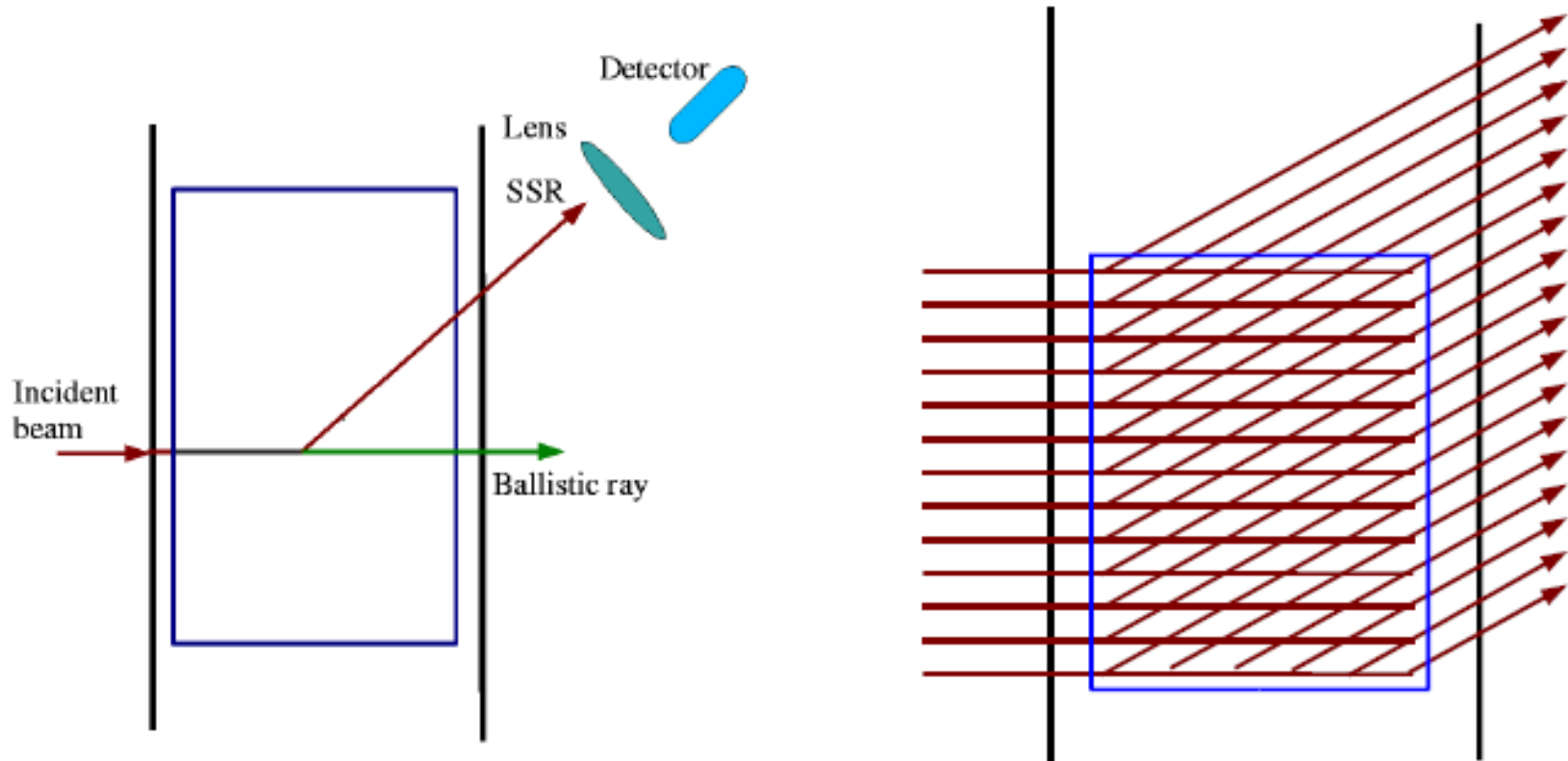


Strong Scattering Regime: Diffuse Optical Tomography



- Many source-detector pairs
- Severely ill-posed IP
- Nonlinear IP

Mesoscopic Scattering Regime: Single-Scattering Tomography



SSOT And Other Modalities

	Linear IP	IP is MILDLY Ill-posed	Single projection	Reflection geometry	Nonionizing radiation	Quantitative images	New contrast mechanisms
X-ray CT	Y (if $E=\text{const}$)	Y	N	N	N	Y	N
DOT	N	N	Y	Y	Y	?	Y
SSOT	Y	Y	Y	Y	Y	Y	Y

The Broken-Ray Integral Transform

(a) $\mu_s = \text{const}$ (and is known)

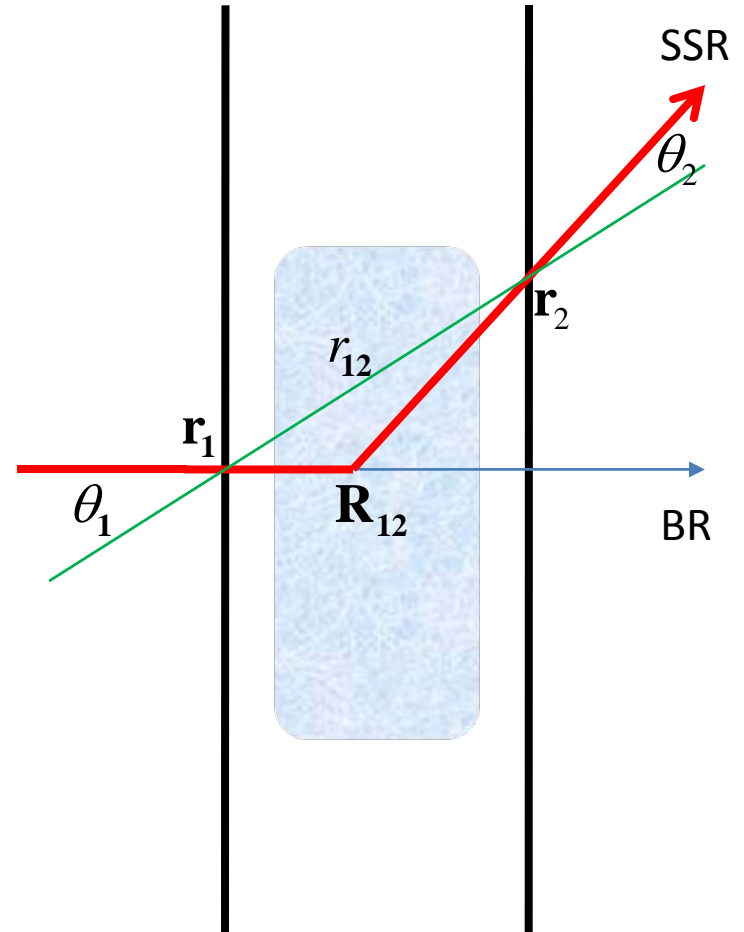
$$\int_{\text{SSR}(\mathbf{r}_1, \hat{\mathbf{s}}_1; \mathbf{r}_2, \hat{\mathbf{s}}_2)} \mu_t[\mathbf{r}(\ell)] d\ell = \phi(\mathbf{r}_1, \hat{\mathbf{s}}_1; \mathbf{r}_2, \hat{\mathbf{s}}_2)$$

(b) $\mu_s \neq \text{const}$ (and is unknown)

$$\int_{\text{SSR}(\mathbf{r}_1, \hat{\mathbf{s}}_1; \mathbf{r}_2, \hat{\mathbf{s}}_2)} \mu_t[\mathbf{r}(\ell)] d\ell - \ln \left[\frac{\mu_s(\mathbf{R}_{12})}{\langle \mu_s \rangle} \right] = \phi(\mathbf{r}_1, \hat{\mathbf{s}}_1; \mathbf{r}_2, \hat{\mathbf{s}}_2)$$

The measurable data function:

$$\phi(\mathbf{r}_1, \hat{\mathbf{s}}_1; \mathbf{r}_2, \hat{\mathbf{s}}_2) = -\ln \left[\frac{r_{12} \sin \theta_1 \sin \theta_2}{\langle \mu_s \rangle A(\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2)} \frac{I_{\text{measured}}}{I_{\text{incident}}} \right]$$



SIMULATIONS

- Forward model based on the RTE
- Isotropic scattering
- FULL ACCOUNT OF MULTIPLE SCATTERING
- BOUNDARY CONDITIONS SATISFIED EXACTLY
- 3D integral equation for density discretized on a rectangular grid
- Direct inversion of a well-posed square matrix
- Mathematical details on next page...

$$[\hat{\mathbf{s}} \cdot \nabla + \underbrace{\mu_a(\mathbf{r}) + \mu_s(\mathbf{r})}_{\mu_t(\mathbf{r})}] I(\mathbf{r}, \hat{\mathbf{s}}) = \mu_s(\mathbf{r}) \int \underbrace{\frac{1}{4\pi}}_{A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \frac{1}{4\pi} = \text{const}} I(\mathbf{r}, \hat{\mathbf{s}}') d^2 \hat{\mathbf{s}}'$$

$$u(\mathbf{r}) = \int I(\mathbf{r}, \hat{\mathbf{s}}) d^2 \hat{\mathbf{s}}$$

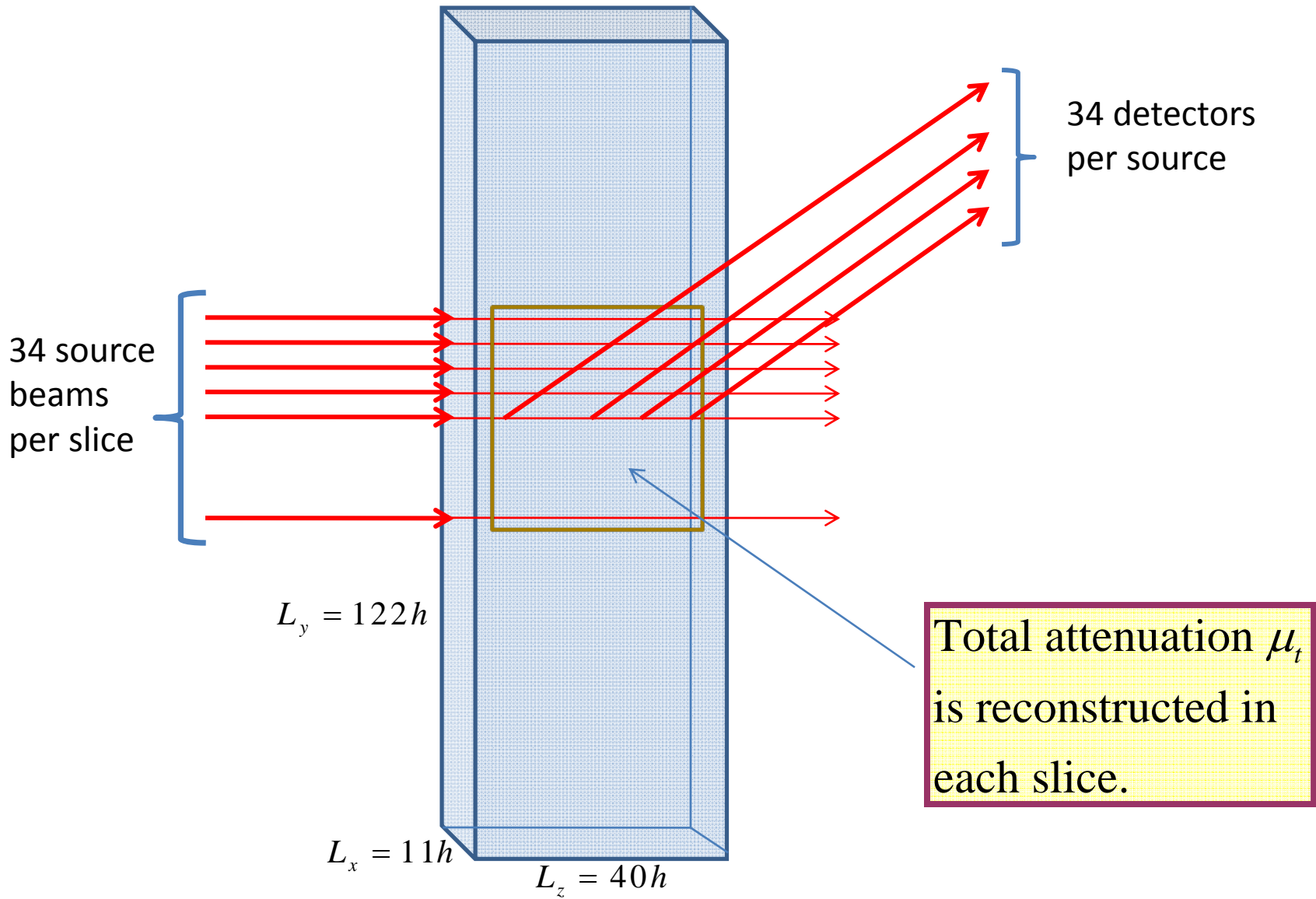
$$u(\mathbf{r}) = u_b(\mathbf{r}) + \int g_b(\mathbf{r}, \mathbf{r}') \frac{\mu_s(\mathbf{r}')}{4\pi} u(\mathbf{r}') d^3 r'$$

$$I(\mathbf{r}, \hat{\mathbf{s}}) = I_b(\mathbf{r}, \hat{\mathbf{s}}) + \int G_b(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}', \hat{\mathbf{s}}') \frac{\mu_s(\mathbf{r}')}{4\pi} u(\mathbf{r}') d^3 r' d^2 \hat{\mathbf{s}}'$$

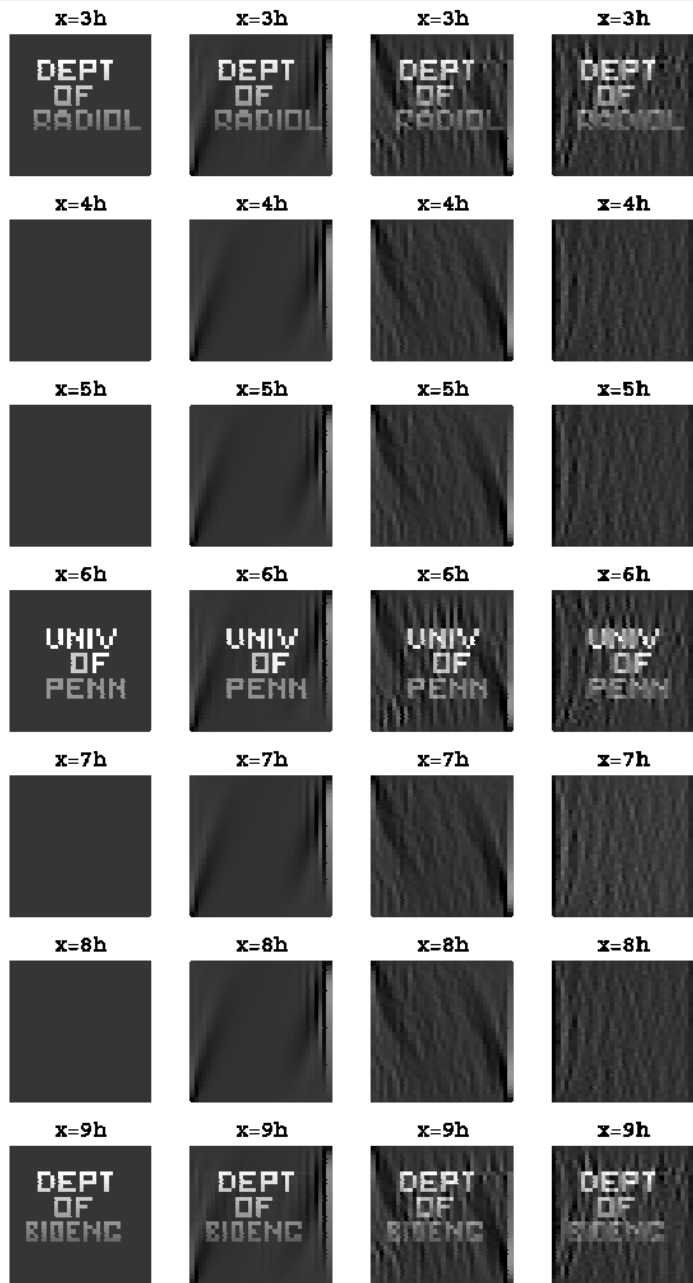
$$G_b(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}', \hat{\mathbf{s}}') = g_b(\mathbf{r}, \mathbf{r}') \delta(\hat{\mathbf{u}}(\mathbf{r} - \mathbf{r}') - \hat{\mathbf{s}}') \delta(\hat{\mathbf{s}} - \hat{\mathbf{s}}')$$

$$g_b(\mathbf{r}, \mathbf{r}') = \int G_b(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}', \hat{\mathbf{s}}') d^2 \hat{\mathbf{s}} d^2 \hat{\mathbf{s}}' =$$

$$= \frac{1}{|\mathbf{r} - \mathbf{r}'|^2} \exp \left[- \int_0^{|\mathbf{r} - \mathbf{r}'|} \mu_t(\mathbf{r}' + \ell \hat{\mathbf{u}}(\mathbf{r} - \mathbf{r}')) d\ell \right]$$



Model $n = 0$ $n = 1\%$ $n = 3\%$ ← Noise levels → Model $n = 0$ $n = 1\%$ $n = 3\%$



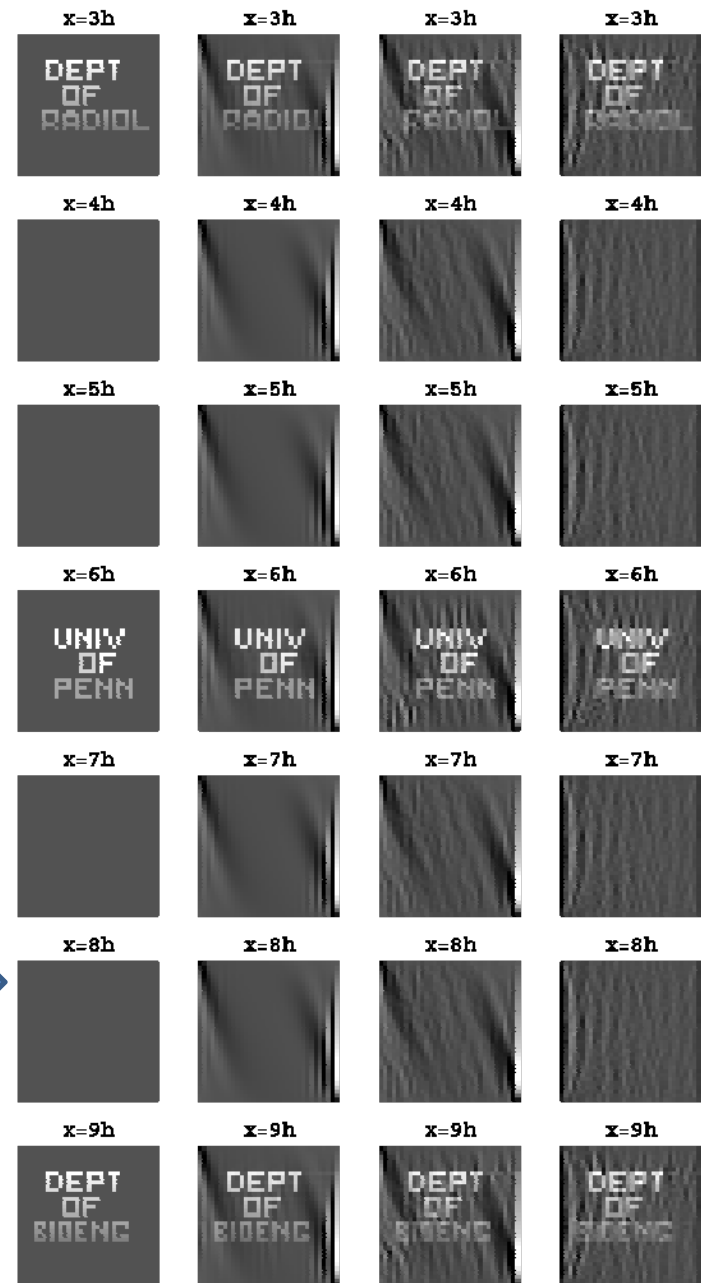
$$\mu_s h = 0.04$$

$$\mu_s L_z = 1.6$$

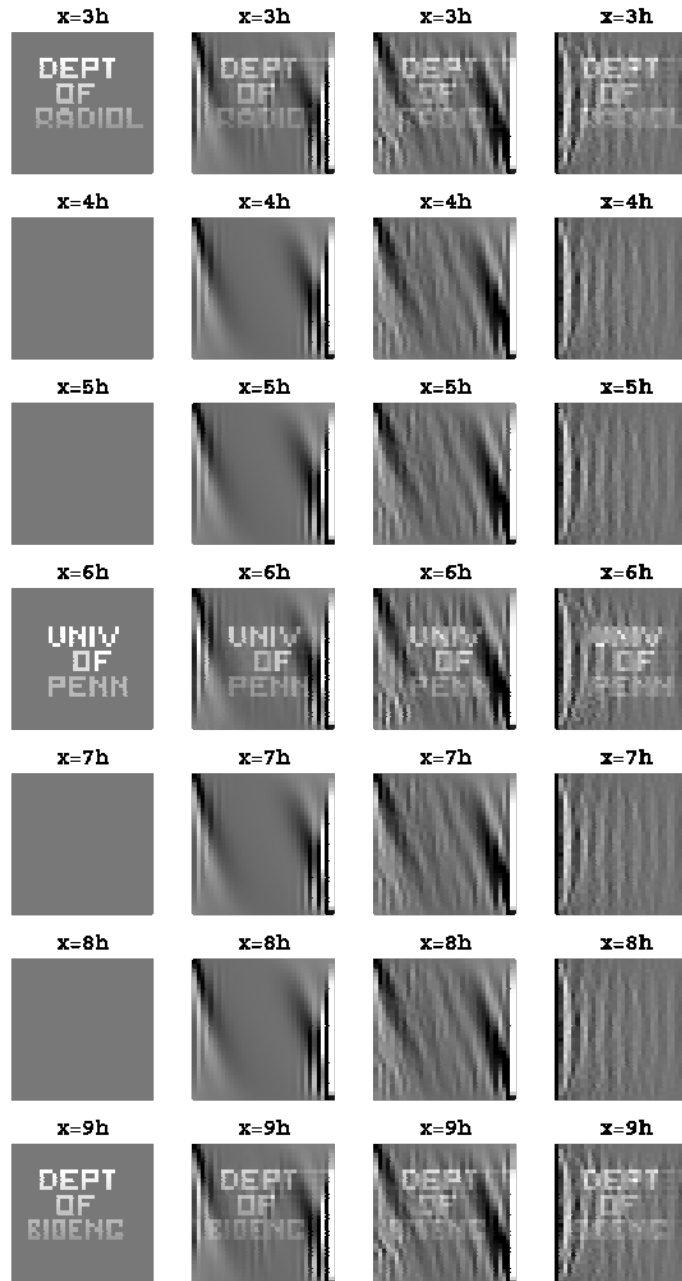
ABSORPTION
Background:
 $\mu_a h = 0.01$
Targets:
 $0.06 < \mu_a h < 0.2$

$$\mu_s h = 0.08$$

$$\mu_s L_z = 3.2$$



Model $n = 0$ $n = 1\%$ $n = 3\%$

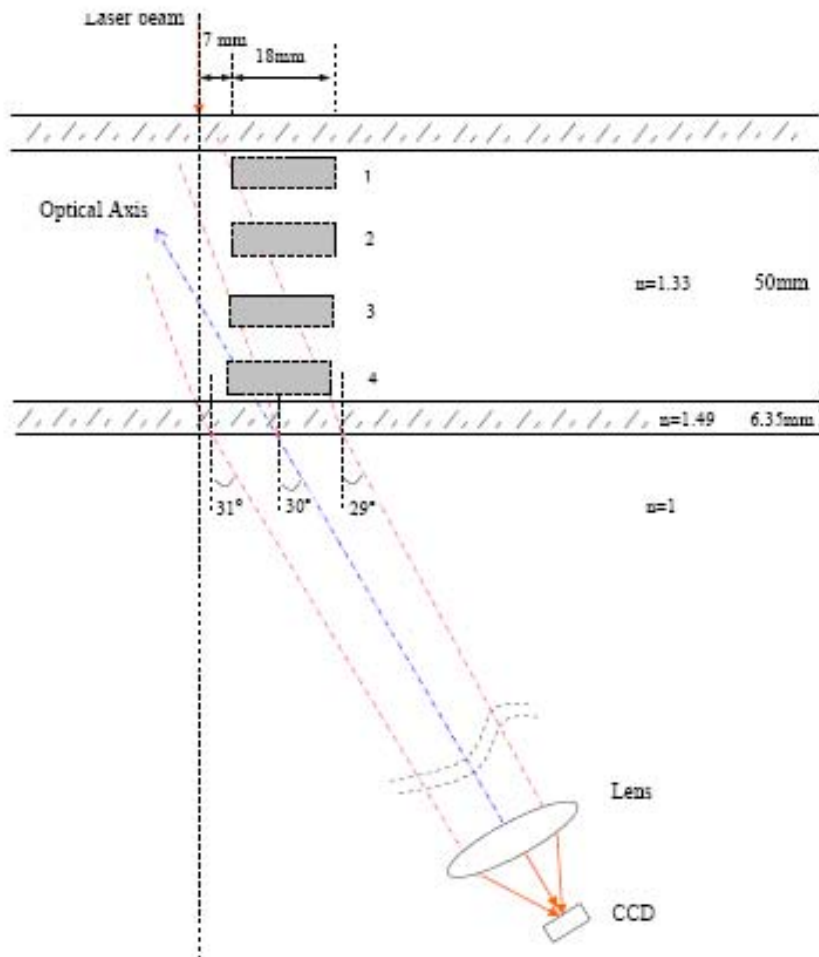


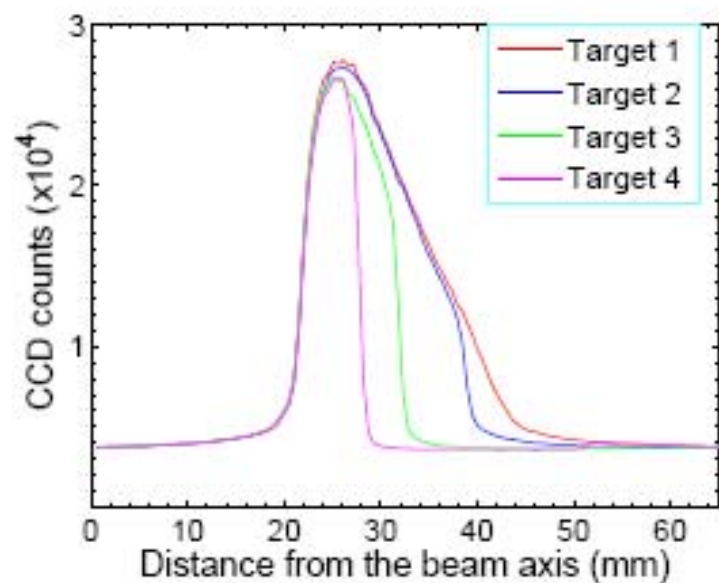
$$\mu_s = 0.16h^{-1}$$
$$\mu_s L_z = 6.4$$

SUMMARY

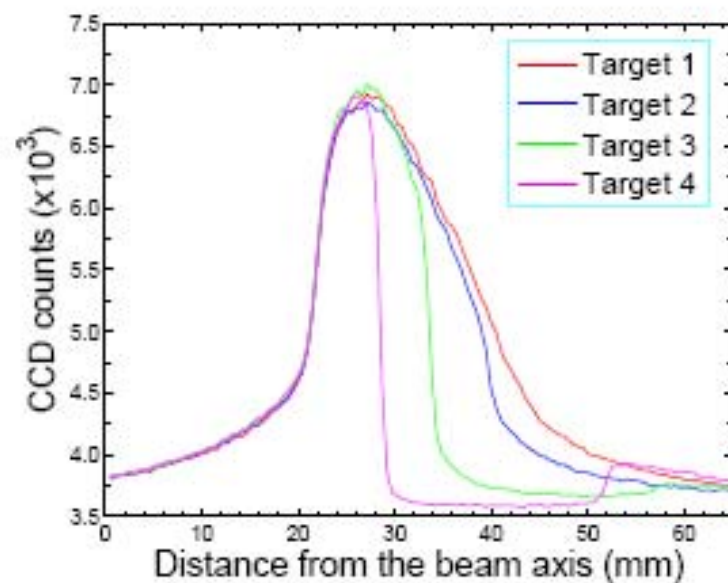
- SSOT allows accurate quantitative reconstruction of the attenuation function
- Ill-posedness of the inverse problem is very mild
- Tomographic imaging is feasible up to about six scattering lengths, with the noise-to-signal level of about 3%

Preliminary Experiment





(a)



(b)

Figure 6: Experimental measurements of the specific intensity as a function of the exit position on the slab surface for intralipid concentrations 0.02% (a) and 0.04% (b).