Optical Diffusion Tomography with Large Data Sets

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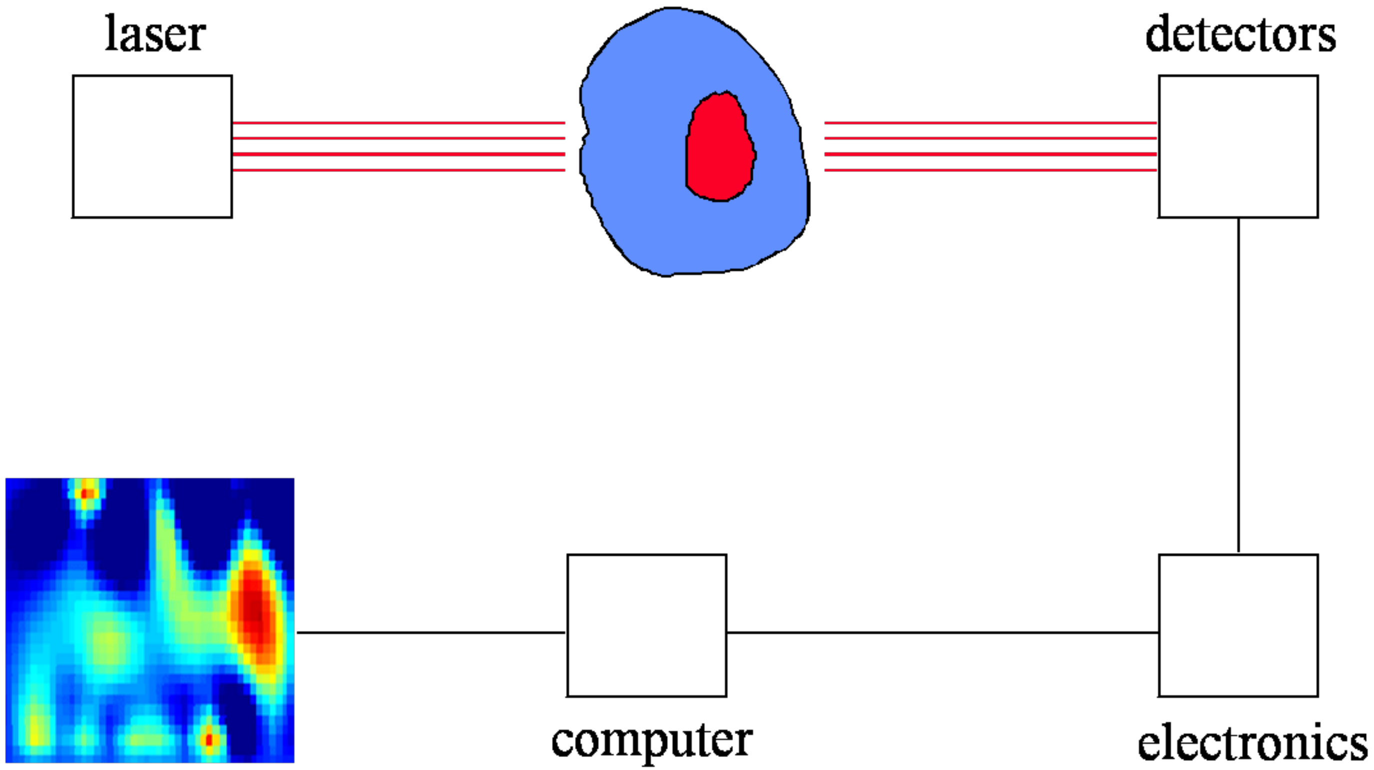
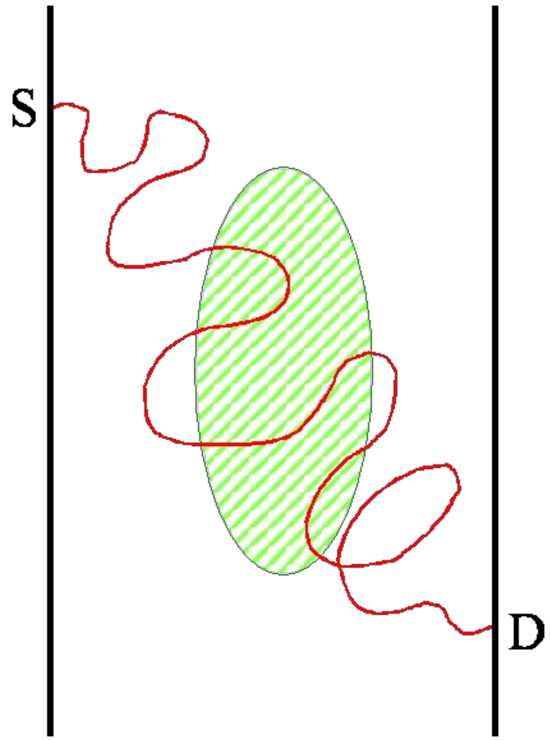
Optical Tomography



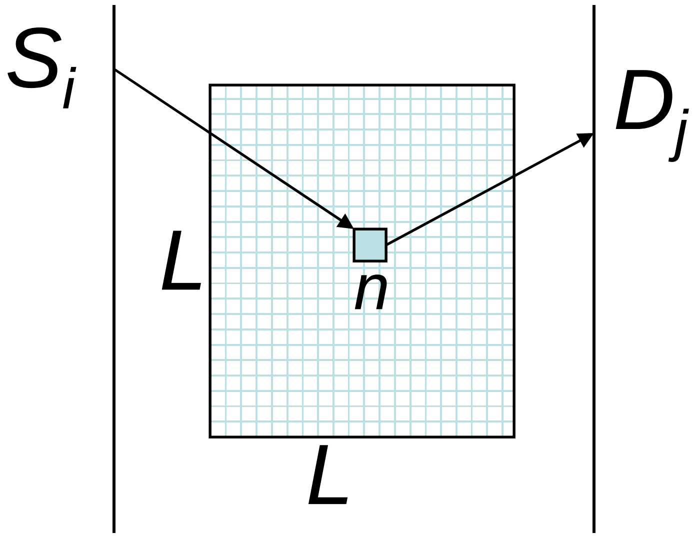
Inverse problem



- Ill-posed
- Nonlinear

Problem: Given measurements from multiple source-detector pairs, reconstruct the spatial distribution of the optical absorption and scattering (or “diffusion”) coefficients.

Size of the Data Set and Complexity of the Problem



ϕ_{ij} (one data point)

N_s - number of sources

N_d -number of detectors

$N = N_s N_d$ – total number of data points

L^3 – number of volume elements

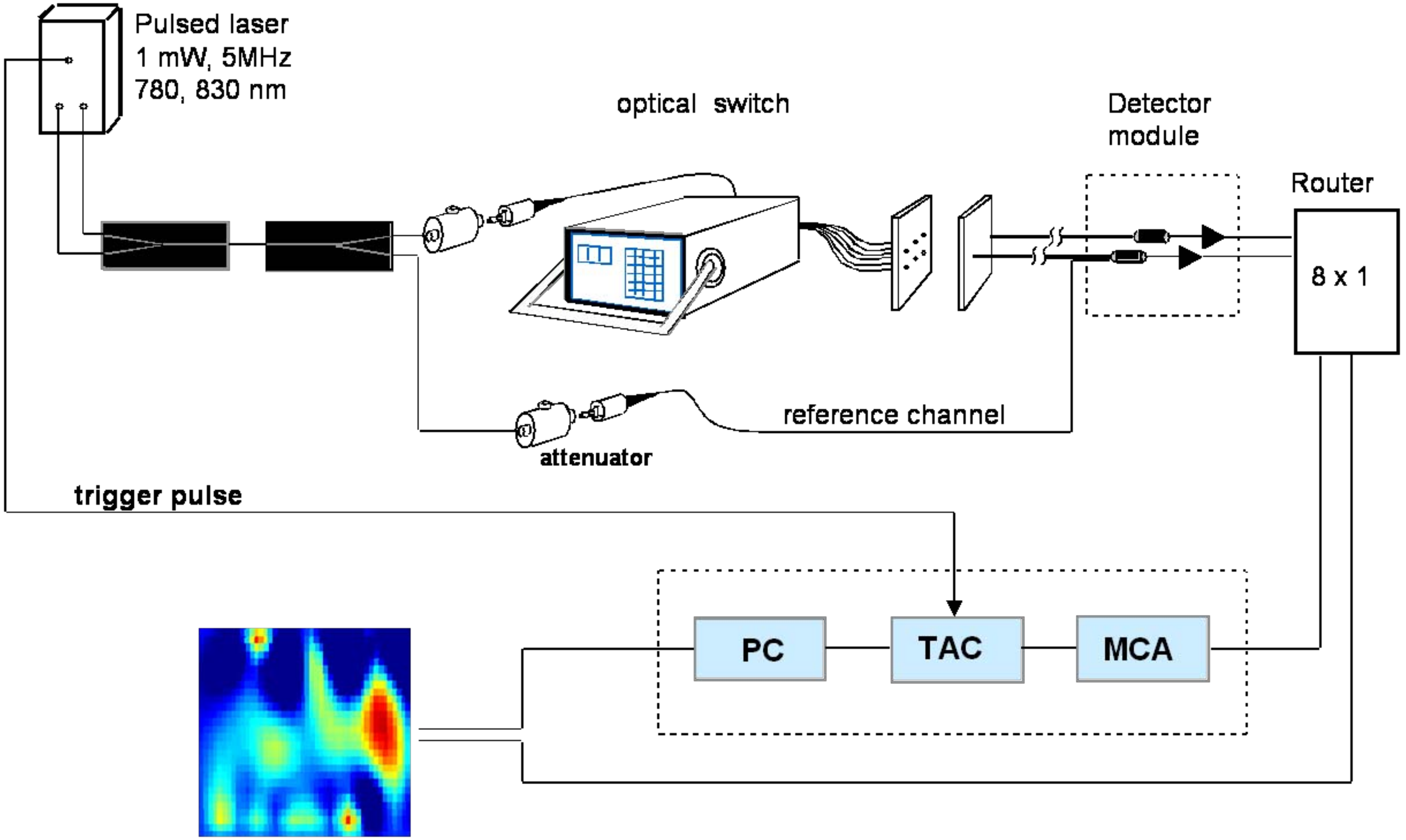
$$\phi_{ij} = \sum_j \Gamma_{ij,n} \delta\alpha_n$$

$$i = 1, \ldots, N_s$$

$$j = 1, \ldots, N_d$$

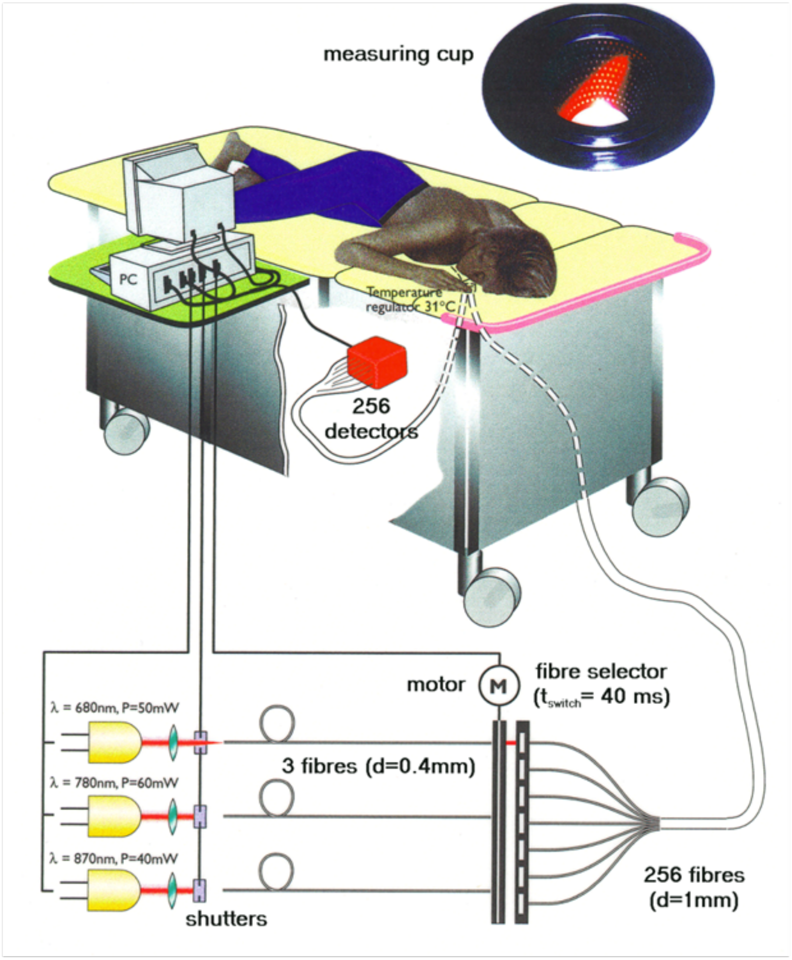
$$n = 1, \ldots, L^3$$

First generation Penn Scanner (~1995)



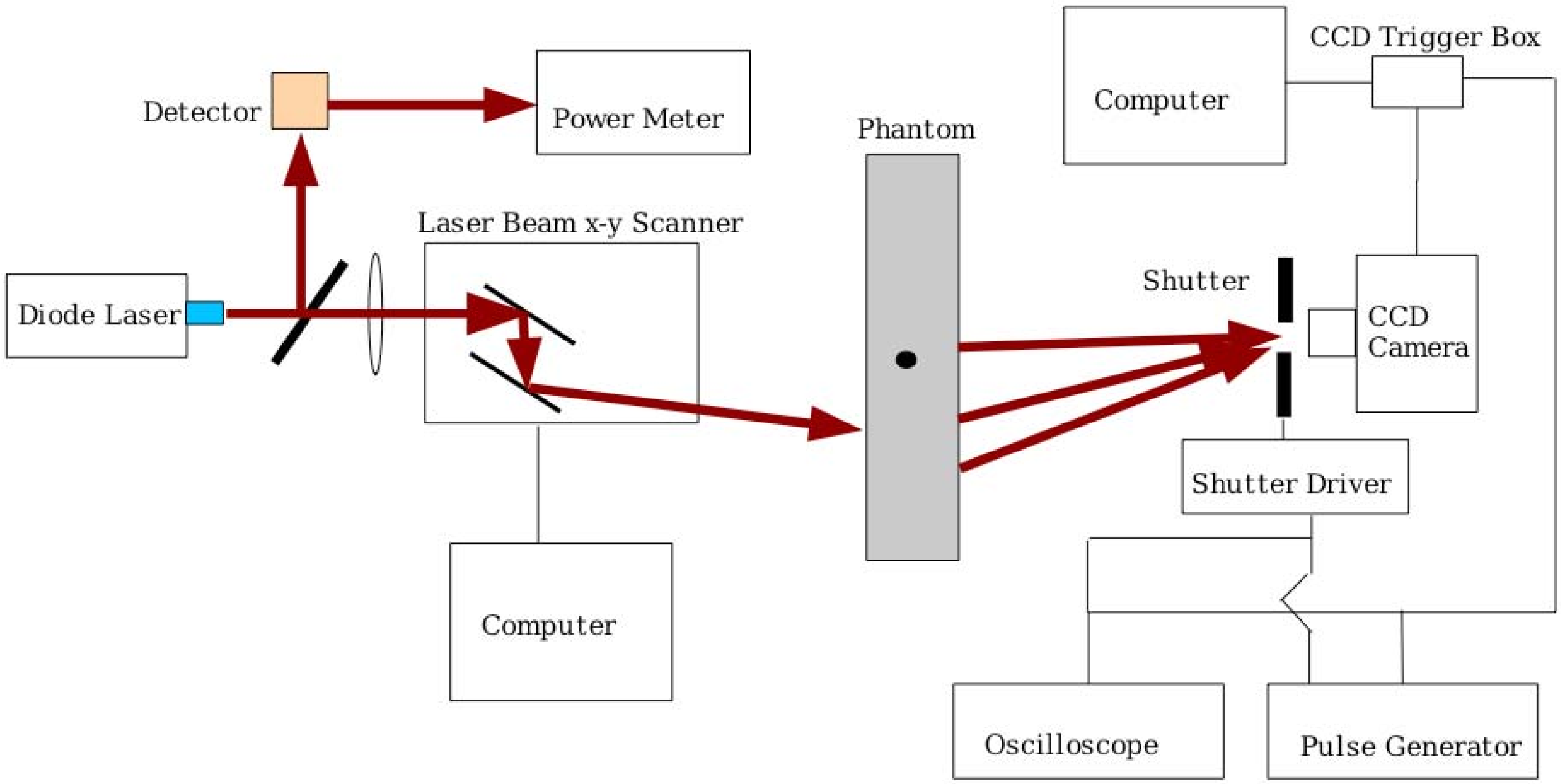
~100 source-detector pairs

Philips Scanner (~1998)



~10^5 source-detector pairs

Noncontact Imager (2005)



10^8 - 10^{10} source detector pairs

Analytical vs Numerical Image Reconstruction Methods

	Analytical	Numerical
Advantages	Arbitrarily high volume discretization Computational efficiency	Generality Large dynamic range of detectors is not required
Disadvantages	Requires special geometry, very large field of view, and high dynamic range of detectors…	Difficult to achieve high volume discretization Large computational complexity

This talk is about numerical methods

Linearization (*D*=const)

$$I_0(\mathbf{r}_d,\mathbf{r}_s) = C_d(\mathbf{r}_d)C_s(\mathbf{r}_s)(1+\ell^*/\ell)^2 G_0(\mathbf{r}_d,\mathbf{r}_s)$$

$$I(\mathbf{r}_d,\mathbf{r}_s) = C_d(\mathbf{r}_d)C_s(\mathbf{r}_s)(1+\ell^*/\ell)^2 G(\mathbf{r}_d,\mathbf{r}_s)$$

G – Green's function for the diffusion equation

C – coupling coefficients

I – measured intensity

$$I(\mathbf{r}_d,\mathbf{r}_s)/I_0(\mathbf{r}_d,\mathbf{r}_s) = G(\mathbf{r}_d,\mathbf{r}_s)/G_0(\mathbf{r}_d,\mathbf{r}_s)$$

Mean-filed approximation for *G*

$$G(\mathbf{r}_d,\mathbf{r}_s) = G_0^2(\mathbf{r}_d,\mathbf{r}_s)/[G_0(\mathbf{r}_d,\mathbf{r}_s) + \int G_0(\mathbf{r}_d,\mathbf{r})\delta\alpha(\mathbf{r})G_0(\mathbf{r},\mathbf{r}_s)d^3r]$$

$$\int G_0(\mathbf{r}_d,\mathbf{r})\delta\alpha(\mathbf{r})G_0(\mathbf{r},\mathbf{r}_s)d^3r = \phi(\mathbf{r}_d,\mathbf{r}_s)$$

⇐ Measurable data function

Discretization

$$\Gamma\delta\alpha = \phi$$

$N \times M$ matrix

vector of length N

vector of length M

$N \gg M$

Size of the Problem

- 20,000 sources per detector
- 29x29=841 source
- N=1.7e7 data points
- M=15x51x51=3.9e4 volume voxels

Computational complexity

$$\mathrm{SVD}: \quad \delta\alpha^{+} = \Gamma^{+}\phi$$

$$\Gamma^{+} = (\Gamma^{*}\Gamma)^{-1}\Gamma^{*}$$

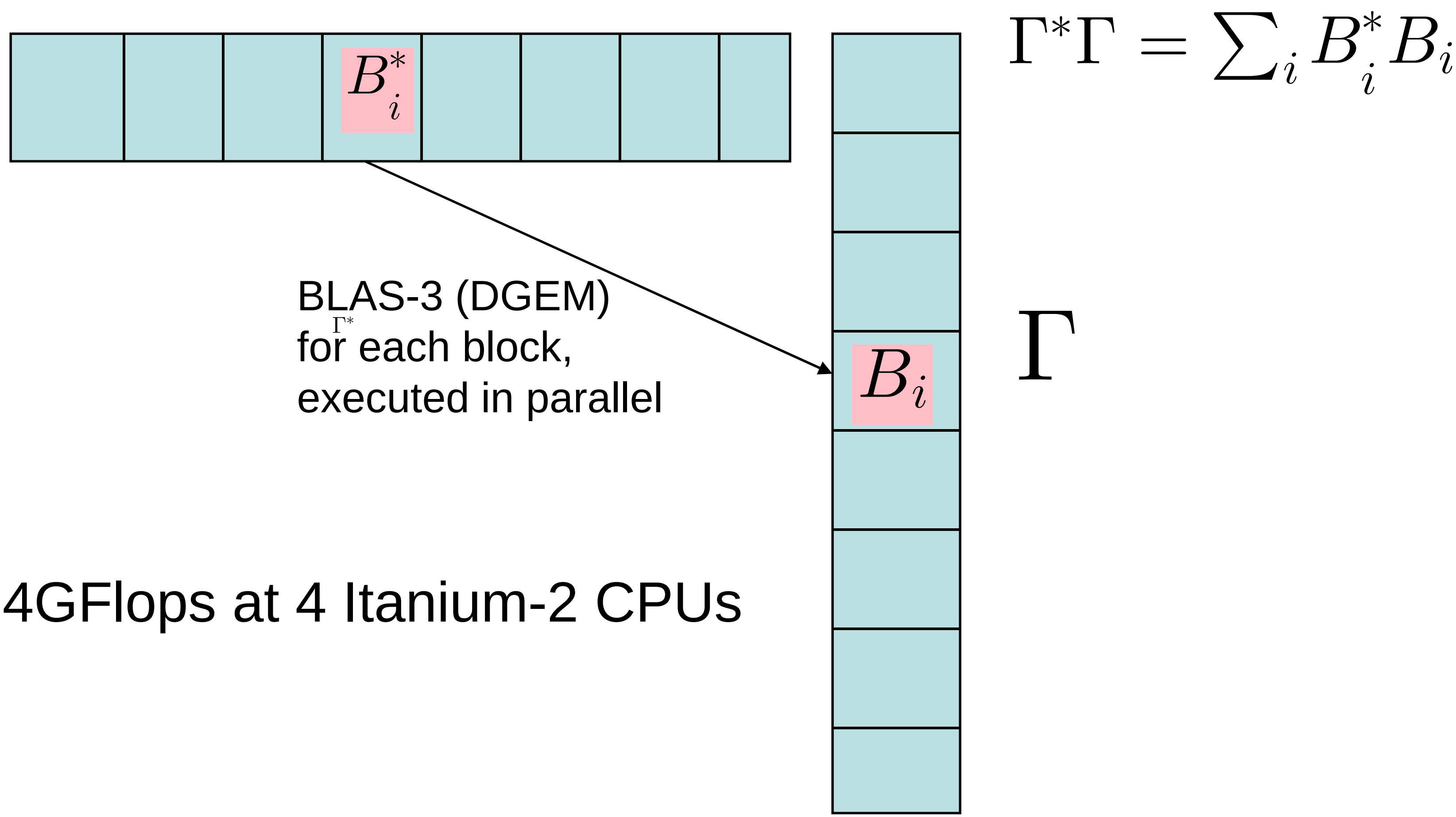
SVD complexity: $aM^2N + bN^3$

$$\delta\alpha^{+} = (\Gamma^{*}\Gamma)^{-1} \; (\Gamma^{*}\phi)$$

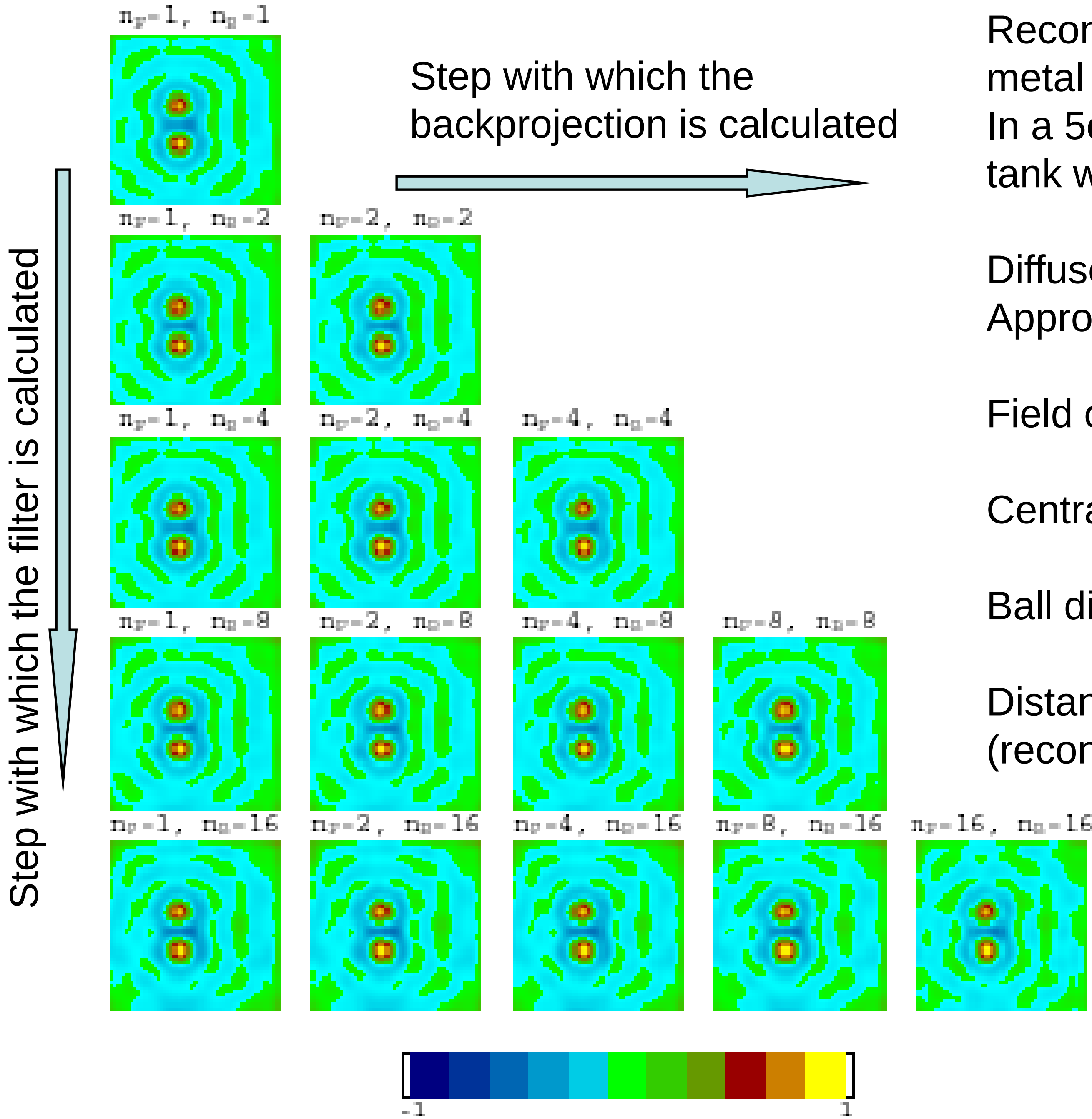
Idea

- Compute the “backprojection” $\Gamma^* \phi$ exactly
- Compute the “filter” $(\Gamma^* \Gamma)^{-1}$ approximately
- This is not the same as binning (presumably, better)
- Allows to average out noise in the data (assuming the noise is non-correlated)
- Approximate method is compared to a calculation with an exact “filter”

Fast Matrix-Matrix Multiplication



24GFlops at 4 Itanium-2 CPUs



Reconstruction of two black metal balls suspended
In a 5cm thick plane-parallel tank with intrelipid solution

Diffuse wavelength
Approximately 10cm

Field of view 14cmx14cm

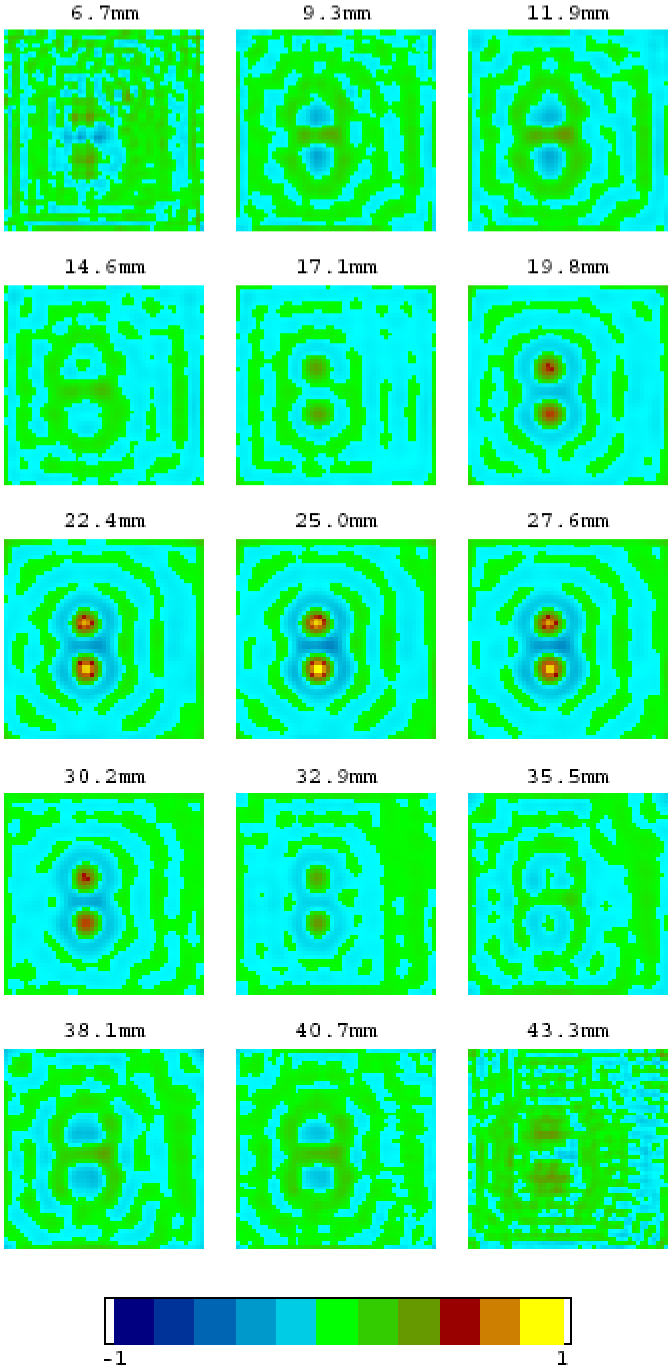
Central slice shown

Ball diameter 8mm;

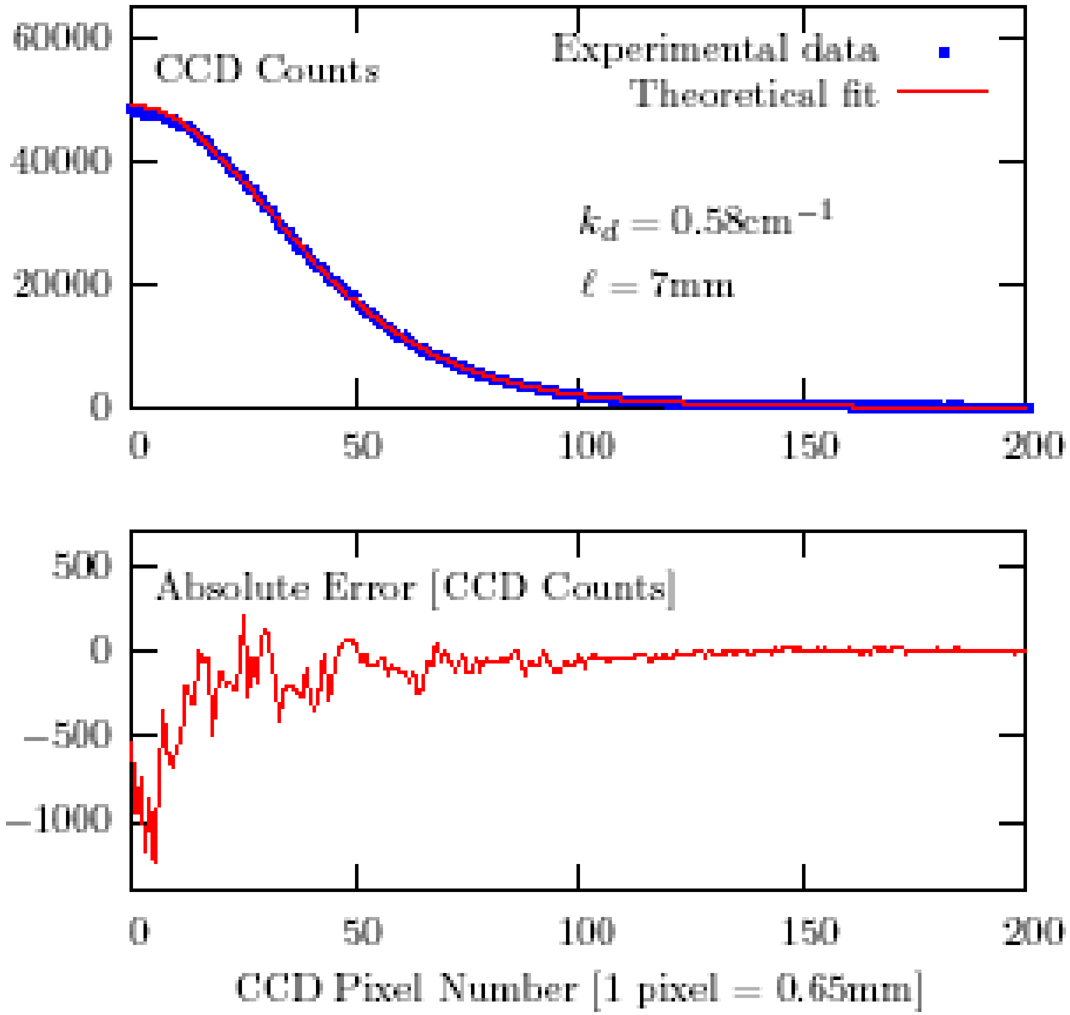
Distance between balls 30mm
(reconstructed correctly)

Fifteen slices drawn parallel to the slab surface at equal separations

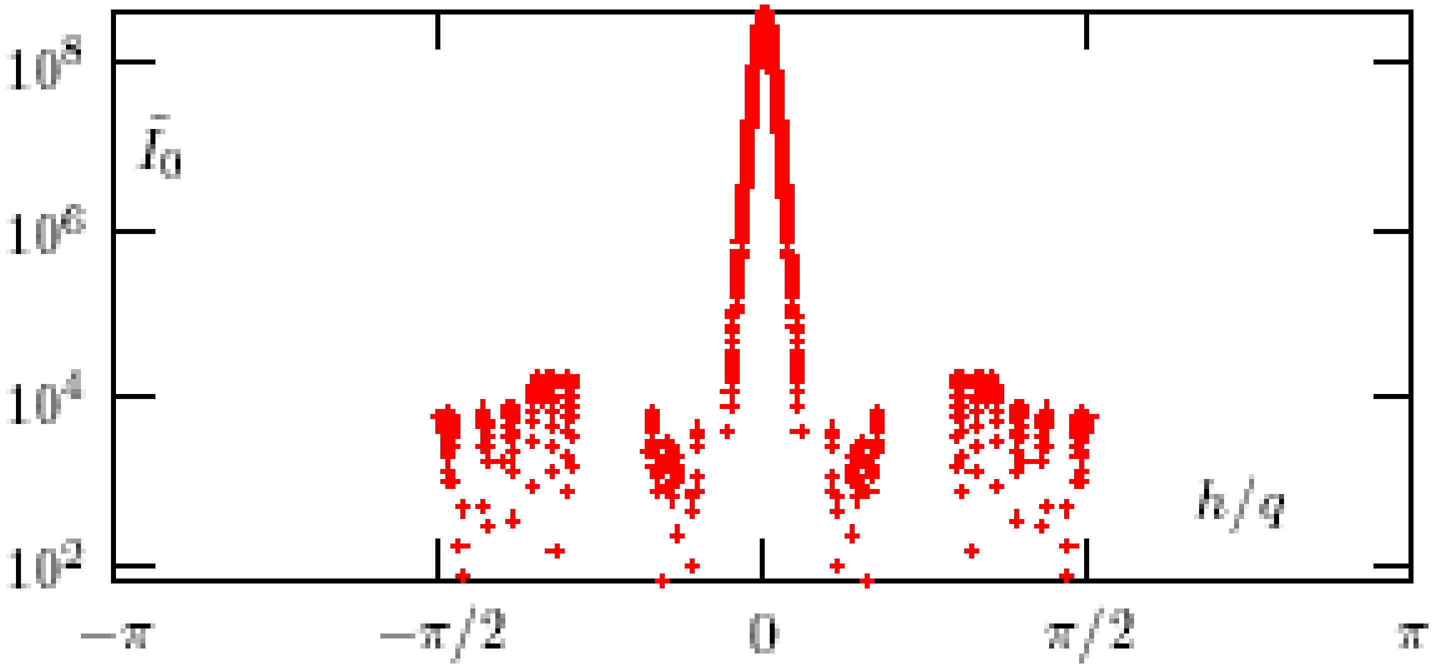
2.5mm between slices



Noise in the Data



(Spatial) Fourier Spectrum of the Transmitted Intensity



h is the step on the surface of the slab corresponding to 1 CCD pixel (*h* = 6.5mm)

Conclusions

- It is possible to compute numerical SVD with > 1e7 data points (more than 2 orders of magnitude more than currently being used)
- This computation can be very significantly accelerated by the application of approximate numerical procedure discussed in this talk
- The potential benefits of such large data sets are

  (i) Higher spatial resolution

  (ii) Better noise tolerance
- However, with our current apparatus, these advantages can not be confirmed due to limitations specific to our experiment…

Note on Analytical Methods:

- Not reported in this talk
- MUCH faster
- Similar image quality
- A paper is in press in Optics Letters (2005)

Published as

Z. Wang, G.Y.Panasyuk, V.A.Markel and J.C.Schotland , “Experimental demonstration of an analytic method for image reconstruction in optical tomography with large data sets,” *Optics Letters* **30**(24), 3338-3340 (2005).