Green's function of the RTE for a point unidirectional source in infinite homogeneous medium. The obtained solutions depend on eigenvectors and eigenvalues of certain tridiagonal matrices which, in turn, depend only on the form of the phase function. After these quantities are computed numerically, the dependence of solutions on the position and direction of propagation is found analytically.

The approach developed below can be applied to the Boltzmann equation describing transport of any waves or particles that move with a fixed absolute velocity c but can change direction as a result of collisions or scattering events. To be more specific, we will assume that the RTE describes propagation of light in a multiply scattering medium. In this case c is the average velocity of light in this medium (set to unity everywhere below) and the quantity of interest is the specific intensity $I(\mathbf{r}, \hat{\mathbf{s}})$ at the point $\mathbf{r}$ and in the direction specified by the unit vector $\hat{\mathbf{s}}$. The RTE has the form

$$\hat{\mathbf{s}} \cdot \nabla I + (\mu_a + \mu_s) I = \mu_s \hat{A} I + \varepsilon. \tag{1}$$

Here μ_a and μ_s are the absorption and scattering coefficients, respectively, $\hat{A}$ is an integral operator defined by

$$\hat{A} I(\mathbf{r}, \hat{\mathbf{s}}) = \int A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') I(\mathbf{r}, \hat{\mathbf{s}}') \, \mathrm{d}^2 \hat{\mathbf{s}}', \tag{2}$$

$A(\hat{\mathbf{s}}, \hat{\mathbf{s}}')$ is the phase function and, finally, $\varepsilon = \varepsilon(\mathbf{r}, \hat{\mathbf{s}})$ is the source. We limit consideration to the case when the phase function depends only on the angle between $\hat{\mathbf{s}}$ and $\hat{\mathbf{s}}'$: $A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = f(\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}')$. This fundamental assumption is often used and corresponds to scattering by spherically symmetrical particles. We also require that the phase function is normalized by the condition $\int A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \, \mathrm{d}^2 \hat{\mathbf{s}}' = 1$.

We start with expressing all position-dependent functions as Fourier integrals, according to

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \int \tilde{I}(\mathbf{q}, \hat{\mathbf{s}}) \exp(\mathrm{i}\mathbf{q} \cdot \mathbf{r}) \, \mathrm{d}^3 q, \tag{3}$$

$$\varepsilon(\mathbf{r}, \hat{\mathbf{s}}) = \int \tilde{\varepsilon}(\mathbf{q}, \hat{\mathbf{s}}) \exp(\mathrm{i}\mathbf{q} \cdot \mathbf{r}) \, \mathrm{d}^3 q. \tag{4}$$

Substituting (3) and (4) into (1), we obtain

$$(\mathrm{i}\mathbf{q} \cdot \hat{\mathbf{s}} + \mu_t) \tilde{I} = \mu_s \hat{A} \tilde{I} + \tilde{\varepsilon}, \tag{5}$$

where we have introduced the notation $\mu_t = \mu_a + \mu_s$.

Next we expand all angular-dependent quantities in spherical harmonics which are defined in the reference frame whose z-axis coincides with the direction of the vector $\mathbf{q}$. We denote such functions as $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}})$ and write

$$\tilde{I}(\mathbf{q}, \hat{\mathbf{s}}) = \sum_{lm} F_{lm}(\mathbf{q}) Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}}), \tag{6}$$

$$\tilde{\varepsilon}(\mathbf{q}, \hat{\mathbf{s}}) = \sum_{lm} E_{lm}(\mathbf{q}) Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}}), \tag{7}$$

$$A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_l Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}}) Y^*_{lm}(\hat{\mathbf{s}}'; \hat{\mathbf{q}}). \tag{8}$$

This expansion is crucial to the theoretical development presented in this letter. It differs from the usual spherical harmonic expansion in the fact that the spherical harmonics used here are defined not in the laboratory reference frame but in a frame which is rotated so that the z-axis is always aligned with the vector $\mathbf{q}$. This approach will lead to significant simplifications as shown below.

Note that there are infinitely many rotations of the laboratory frame that result in the new (rotated) z-axis being aligned with $\mathbf{q}$. It is sufficient to choose the Euler angles of the rotation, α, β and γ, so that $\alpha = \phi$, $\beta = \theta$ and $\gamma = 0$, where ϕ and θ are the polar angles of the vector $\mathbf{q}$ in the laboratory frame. Then the functions $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}})$ can be expressed as linear combinations of spherical harmonics $Y_{lm}(\hat{\mathbf{s}})$ defined in the laboratory frame with the use of Wigner D-functions according to $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}}) = \sum_{m'=-l}^{l} D^{l}_{m'm}(\phi, \theta, 0) Y_{lm'}(\hat{\mathbf{s}})$.

Note also that the coefficients A_l in (8) do not depend on $\mathbf{q}$. From the normalization condition $\int A(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\, \mathrm{d}^2\hat{\mathbf{s}}' = 1$ it follows that $A_0 = 1$. In the case of isotropic scattering, the phase function is constant and $A_l = \delta_{l0}$ where $\delta_{ll'}$ is the Kronecker delta. It is in this case that the analytical solution mentioned above can be obtained. In general, since $A_l = \int P_l(\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}') A(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\, \mathrm{d}^2\hat{\mathbf{s}}'$ where $P_l(x)$ are Legendre polynomials, and taking into account that $|P_l(x)| \leqslant 1$, it can be shown that $|A_l| \leqslant 1\ \forall l$. Our goal is to obtain the solution to the RTE for coefficients A_l satisfying the above inequality, but otherwise arbitrary.

Upon substitution of expansions (6)–(8) into (5), we find that the coefficients $F_{lm}(\mathbf{q})$ satisfy

$$\mathrm{i}q \sum_{l'm'} R_{lm,l'm'} F_{l'm'}(\mathbf{q}) + \sigma_l F_{lm}(\mathbf{q}) = E_{lm}(\mathbf{q}), \tag{9}$$

where

$$\sigma_l = \mu_{\mathrm{a}} + \mu_{\mathrm{s}}(1 - A_l), \tag{10}$$

$$R_{lm,l'm'} = \int \hat{\mathbf{s}} \cdot \hat{\mathbf{q}} Y^*_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}}) Y_{l'm'}(\hat{\mathbf{s}}; \hat{\mathbf{q}})\, \mathrm{d}^2\hat{\mathbf{s}}. \tag{11}$$

Integration according to (11) results in

$$R_{lm,l'm'} = \delta_{mm'}[b_l(m)\delta_{l'=l-1} + b_{l+1}(m)\delta_{l'=l+1}], \tag{12}$$

where

$$b_l(m) = b_l(|m|) = \sqrt{(l+m)(l-m)/(2l+1)(2l-1)}. \tag{13}$$

Now we make use of the fact that the matrix R defined in (12) is diagonal in m and m'. For every value of $\mathbf{q}$ and every value of $m = 0, \pm 1, \pm 2, \ldots$ we define infinite-dimensional vectors $|F(\mathbf{q}, m)\rangle$ and $|E(\mathbf{q}, m)\rangle$ with components $\langle l|F(\mathbf{q}, m)\rangle = F_{lm}(\mathbf{q})$ and $\langle l|E(\mathbf{q}, m)\rangle = E_{lm}(\mathbf{q})$, $l \geqslant |m|$. We also define $\mathbf{q}$-independent matrices $R(m)$ and $S(m)$ with elements $\langle l|R(m)|l'\rangle = R_{lm,l'm}$ and $\langle l|S(m)|l'\rangle = \sigma_l \delta_{ll'}$, $l, l' \geqslant |m|$ and rewrite (9) in operator form as

$$\mathrm{i}q R(m)|F(\mathbf{q}, m)\rangle + S(m)|F(\mathbf{q}, m)\rangle = |E(\mathbf{q}, m)\rangle. \tag{14}$$

It can be seen that (14) is a set of independent equations parametrized by the variables m and $\mathbf{q}$.

Because of the specific choice of the basis functions $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{q}})$, equation (14) has a simple form. This is manifested in the fact that the matrices $R(m)$ are $\mathbf{q}$ independent and tridiagonal. Now we use these simplifications to find a solution to (14) which is based on diagonalization of several $\mathbf{q}$-independent matrices. It is important to emphasize that the numerical part must be carried out only once, rather than for every value of $\mathbf{q}$ involved in integration according to (3). We notice that the matrix S is non-singular and positive-definite, which follows directly from $|A_l| \leqslant 1$ and $\mu_{\mathrm{a}} > 0$, and introduce the 'square root' of S, $T = \sqrt{S}$. Here T is a diagonal matrix satisfying $TT = S$ with diagonal elements $\sqrt{\sigma_l}$, $l \geqslant |m|$. Then we can rewrite (14) equivalently as

$$\mathrm{i}q W(m)|F'(\mathbf{q}, m)\rangle + |F'(\mathbf{q}, m)\rangle = T^{-1}(m)|E(\mathbf{q}, m)\rangle, \tag{15}$$

where $|F'\rangle = T|F\rangle$ and $W = T^{-1}RT^{-1}$. Now we can solve (15) for $|F'\rangle$ by diagonalizing the tridiagonal symmetrical matrix W, after which $|F\rangle$ can be found from $|F\rangle = T^{-1}|F'\rangle$. Let $|y_n\rangle$

and λ_n be the eigenvectors and eigenvalues of W, respectively. Following the procedure briefly outlined above, we find that the solution to (14) is given by

$$|F(\mathbf{q}, m)\rangle = \sum_n \frac{T^{-1}|y_n(m)\rangle\langle y_n(m)|T^{-1}(m)|E(\mathbf{q}, m)\rangle}{1 + \mathrm{i}q\lambda_n(m)}, \tag{16}$$

or, in components,

$$F_{lm}(\mathbf{q}) = \sum_n \sum_{l'=|m|}^{\infty} \frac{\langle l|y_n(m)\rangle\langle y_n(m)|l'\rangle E_{l'm}(\mathbf{q})}{\sqrt{\sigma_l\sigma_{l'}}[1 + \mathrm{i}q\lambda_n(m)]}. \tag{17}$$

Equations (16) and (17) together with (6) and (3) present the general solution to the RTE with an arbitrary source and phase function. In the important case of a point source the Fourier transformation according to (3) can be carried out analytically. The integration is facilitated by the following fact. If λ is an eigenvalue of W corresponding to the eigenvector with components $\langle l|y\rangle$, then $-\lambda$ is also an eigenvalue corresponding to the eigenvector with components $(-1)^l\langle l|y\rangle$ (a similar property of eigenvalues was found in [10] for a slab geometry and a normally incident plane wave). This can be easily proven by considering the characteristic equation for W which has the form

$$\beta_l\langle l-1|y\rangle + \beta_{l+1}\langle l+1|y\rangle = \lambda\langle l|y\rangle, \tag{18}$$

where $\beta_l = b_l(m)/\sqrt{\sigma_{l-1}\sigma_l}$ ($l = 1, 2, \ldots$) are the elements of the first superdiagonal of W.

Consider a unidirectional point source of the form $\varepsilon = \delta(\mathbf{r}-\mathbf{r}_0)\delta(\hat{\mathbf{s}}-\hat{\mathbf{s}}_0)$. The coefficients $E_{lm}(\mathbf{q})$ are given for the source defined above by $E_{lm}(\mathbf{q}) = (2\pi)^{-3}\exp(-\mathrm{i}\mathbf{q}\cdot\mathbf{r}_0)Y^*_{lm}(\hat{\mathbf{s}}_0; \hat{\mathbf{q}})$. We substitute this expression into (17) and use the property of the eigenvalues and the symmetry of the eigenvectors described above, as well as the properties of Wigner D-functions, to perform integration according to (6) and (3) (details of integration omitted). The result for $I(\mathbf{r}, \hat{\mathbf{s}})$ is

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \sum_{m=-\infty}^{\infty} \sum_{l,l'=|m|}^{\infty} \chi^m_{ll'}(R) Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{R}}) Y^*_{l'm}(\hat{\mathbf{s}}_0; \hat{\mathbf{R}}), \tag{19}$$

where

$$\chi^m_{ll'}(R) = \chi^m_{l'l}(R) = \frac{(-1)^m}{2\pi\sqrt{\sigma_l\sigma_{l'}}} \sum_{M=-\bar{l}}^{\bar{l}} (-1)^M {\sum_n}' \frac{\langle l|y_n(M)\rangle\langle y_n(M)|l'\rangle}{\lambda_n^3(M)}$$
$$\times \sum_{j=0}^{\bar{l}} C^{|l-l'|+2j,0}_{l,M,l',-M} C^{|l-l'|+2j,0}_{l,m,l',-m} k_{|l-l'|+2j}\left[\frac{R}{\lambda_n(M)}\right], \tag{20}$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}_0, \qquad \bar{l} = \min(l, l'). \tag{21}$$

Here $k_n(x) = -\mathrm{i}^n h^{(1)}_n(\mathrm{i}x)$ is the modified spherical Hankel function of the first kind, $C^{j_3,m_3}_{j_1,m_1,j_2,m_2}$ are the Clebsch–Gordan coefficients and $\sum'$ denotes summation over only positive values of λ_n.

Equations (19) and (20) constitute the main result of this letter. The function $I(\mathbf{r}, \hat{\mathbf{s}})$ coincides with the Green's function of the RTE, $G(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}_0, \hat{\mathbf{s}}_0)$, while the functions $\chi^m_{ll'}(R)$ can be viewed as the matrix elements of the Green's function in the basis of spherical harmonics $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{R}})$:

$$\langle lm|G(\mathbf{r}; \mathbf{r}_0)|l'm'\rangle \equiv \int Y^*_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{R}}) G(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}_0, \hat{\mathbf{s}}_0) Y_{l'm'}(\hat{\mathbf{s}}_0; \hat{\mathbf{R}})\, \mathrm{d}^2s\, \mathrm{d}^2s_0 = \chi^m_{ll'}(R)\delta_{mm'}, \tag{22}$$

where we must extend the definition of $\chi^m_{ll'}$ to all values of indices with the understanding that $\chi^m_{ll'} = 0$ if $|m| > \min(l, l')$.

A few comments on the obtained solutions are necessary. First, the functions $Y_{lm}(\hat{\mathbf{s}}; \hat{\mathbf{R}})$ in (19) are spherical harmonics defined in the $\mathbf{q}$-independent laboratory frame with the z axis coinciding with the direction of vector $\mathbf{R}$. The directions of x and y axes are arbitrary. Second, the terms in the summation over M in (20) do not depend on the sign of M (which is also true for the matrix $W(M)$). Third, the diagonality of the matrix element (22) reflects the fact that the Green's function is symmetrical with respect to simultaneous rotation of vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{s}}_0$ around the axis connecting the points $\mathbf{r}$ and $\mathbf{r}_0$. Note that this is not equivalent to cylindrical symmetry since the solution (19) depends, in general, on three spatial coordinates. Finally, the eigenvalues $\lambda_n(M)$ are bounded. It can be verified using Gershgorin's disc theorem that $\lambda_{\max} \leqslant \max_{M=0}^{\infty} \max_{l=|M|}^{\infty} [\beta_{l-1}(M) + \beta_l(M)] \leqslant 1/\mu_a$. For $R\mu_a \gg 1$ we can use $k_j(x) \approx \exp(-x)/x$ and obtain the asymptotic expression

$$\lim_{\mu_a R \gg 1} [\chi_{ll'}^m(R)] = \frac{(-1)^m}{2\pi\sqrt{\sigma_l \sigma_{l'}} R} \sum_{M=-\bar{l}}^{\bar{l}} (-1)^M K_{ll'}^{mM} {\sum_n}' \frac{\langle l|y_n(M)\rangle\langle y_n(M)|l'\rangle}{\lambda_n^2(M)} \exp\left[\frac{-R}{\lambda_n(M)}\right], \tag{23}$$

where

$$K_{ll'}^{mM} = \sum_{j=0}^{\bar{l}} C_{l,M,l',-M}^{|l-l'|+2j,0} C_{l,m,l',-m}^{|l-l'|+2j,0}. \tag{24}$$

Now we consider a few special cases of the obtained formulae. Note that these special cases can be obtained independently of the method developed in this letter and are already known (see, for example, [2] for the spherically symmetrical problem of an isotropic source); they are adduced to validate the obtained formulae and to provide connection to the previous work. First, if $\hat{\mathbf{R}} = \pm\hat{\mathbf{s}}_0$ ('forward' or 'backward' propagation), we have

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \sum_{l,l'=0}^{\infty} (\pm 1)^{l'} \chi_{ll'}^0(R) \frac{\sqrt{(2l+1)(2l'+1)}}{4\pi} P_l(\hat{\mathbf{s}} \cdot \hat{\mathbf{R}}). \tag{25}$$

Further simplifications can be obtained if the source is isotropic, i.e., $\varepsilon = \delta(\mathbf{r} - \mathbf{r}_0)$. Straightforward calculation shows that in this case

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \sum_{l=0}^{\infty} \sqrt{2l+1} \chi_{l0}^0(R) P_l(\hat{\mathbf{s}} \cdot \hat{\mathbf{R}}), \tag{26}$$

where $\chi_{l0}^0(R)$ has a very simple form

$$\chi_{l0}^0(R) = \frac{1}{2\pi\sqrt{\sigma_0 \sigma_l}} {\sum_n}' \frac{\langle l|y_n(0)\rangle\langle y_n(0)|0\rangle}{\lambda_n^3(0)} k_l\left[\frac{R}{\lambda_n(0)}\right]. \tag{27}$$

Only one matrix, namely $W(0)$, must be diagonalized in order to compute χ_{l0}^0. Note that the diffusion approximation (DA) corresponds to leaving only two terms ($l = 0$ and 1) in equation (26).

One can be interested in the energy density and flux, $u(\mathbf{r})$ and $\mathbf{J}(\mathbf{r})$, defined as $u(\mathbf{r}) = \int I(\mathbf{r}, \hat{\mathbf{s}})\, d^2\hat{\mathbf{s}}$ and $\mathbf{J}(\mathbf{r}) = \int \hat{\mathbf{s}} I(\mathbf{r}, \hat{\mathbf{s}})\, d^2\hat{\mathbf{s}}$, respectively. For these two quantities and for the isotropic source, we find

$$u(\mathbf{r}) = \frac{2}{\sigma_0 R} {\sum_n}' \frac{\langle 0|y_n(0)\rangle\langle y_n(0)|0\rangle}{\lambda_n^2(0)} \exp\left[-\frac{R}{\lambda_n(0)}\right], \tag{28}$$

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{R}} \frac{2}{R} {\sum_n}' \frac{\langle 0|y_n(0)\rangle\langle y_n(0)|0\rangle}{\lambda_n(0)} \left[1 + \frac{\lambda_n(0)}{R}\right] \exp\left[-\frac{R}{\lambda_n(0)}\right], \tag{29}$$

where we have also taken into account that $\langle 1|y_n(0)\rangle = \sqrt{3\sigma_0\sigma_1}\lambda_n(0)\langle 0|y_n(0)\rangle$, which follows directly from the characteristic equation (18).

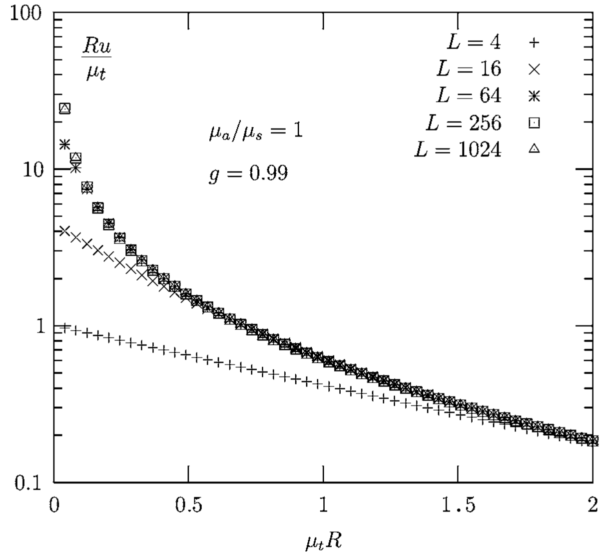

Figure 1. Dimensionless quantity $Ru(R)/\mu_t$ as a function of $\mu_t R$ for different values of L; $\mu_a/\mu_s = 1$ and $g = 0.99$.

Now we briefly discuss implementation of the method. Generally, all infinite series appearing in (19), (25) and (26) must be truncated so that $l, l' \leqslant l_{max}$. The value of l_{max} is determined by the desired angular resolution of the calculated specific intensity. For example, in equations (25) and (26), l_{max} is the maximum order of Legendre polynomials which enter the expansion of the angular dependence of $I(\mathbf{r}, \hat{\mathbf{s}})$. In practice, more terms are required close to the source where the specific intensity is highly peaked, while far from the source the radiation becomes diffuse and account of only a few lower terms may suffice.

The expression for $\chi^m_{ll'}$ (20) contains only finite sums (for finite values of l and l'). However, the coefficients in these sums depend on eigenvectors and eigenvalues of infinite matrices. The latter must be also truncated; we denote the size of truncated matrices as L. As a rough rule, the convergence of results with L can be expected when the elements of the first superdiagonal of W, $\beta_L(m)$, approach their limiting value of $\lim_{l\to\infty} \beta_l(m) = 1/2\mu_t$. However, slower convergence is expected close to the source because, for small values of R, expressions (19), (25) and (26) are more sensitive to small eigenvalues. The convergence of the method is illustrated using the Henyey–Greenstein phase function (figure 1). For this phase function, $A_l = g^l$ where $0 < g < 1$. If g is close to unity (strong forward scattering), the condition that $\beta_L(m)$ approaches its limiting value takes the form $L \gg 1/(1-g)$.

We have plotted the dimensionless function $Ru(R)/\mu_t$ calculated according to expression (26) for an isotropic source for $\mu_a = \mu_s$ and $g = 0.99$ and different values of L. It can be seen that good convergence is obtained for $L = 256$, which is in agreement with the convergence criterion formulated above. It should be emphasized that the computational complexity of diagonalizing tridiagonal matrices $W(M)$ scales as L rather than L^3. Therefore, diagonalization for L up to 10 000 is a relatively simple task on any modern computer.

Fully converged results for the dimensionless combination $Ru(R)/\mu_t$ as a function of $\mu_t R$ and different ratios of μ_a/μ_s and values of g are plotted in figure 2. Here we have used again expression (26) for an isotropic source, the Henyey–Greenstein phase function and the matrix size $L = 1000$ (even though convergence is reached for significantly smaller

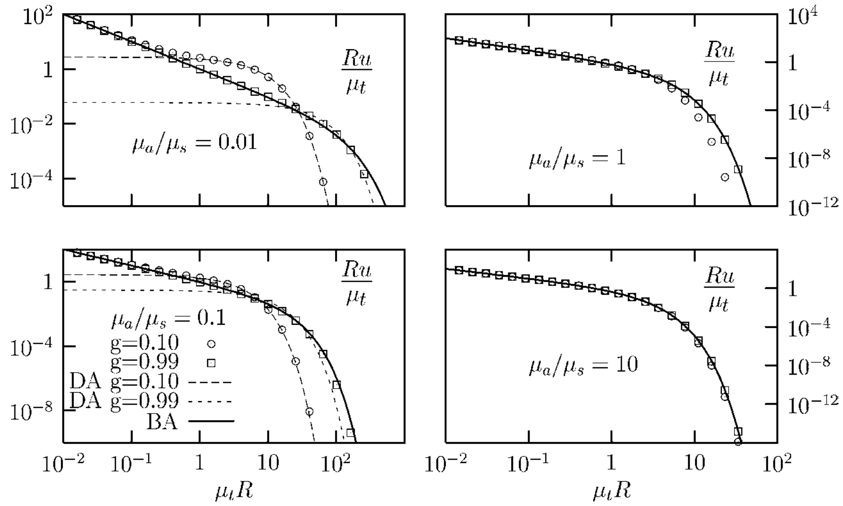

Figure 2. Dimensionless quantity $Ru(R)/\mu_t$ as a function of $\mu_t R$ for different values of μ_a/μ_s and g. The dashed curves show DA and the solid curves BA.

values of L). We also plot in this figure the DA result (dashed curves) and the ballistic approximation (BA) result (solid curve). The DA is given by $u(R) = \exp(-k_{\text{diff}} R)/DR$ where $k_{\text{diff}} = \sqrt{3\mu_a[\mu_a + (1 - A_1)\mu_s]}$ and $D = 1/3[\mu_a + (1 - A_1)\mu_s]$. Note that for the Henyey–Greenstein phase function $A_1 = g$. The BA is obtained by setting $\mu_s = 0$ or, equivalently, $A_l = 1\ \forall l$, in which cases the solution is given by $u(R) = \exp(-\mu_a R)/R^2$. It can be seen that for small values of $\mu_t R$ the propagation is always ballistic. In the cases $\mu_a/\mu_s = 0.01$ and 0.1 the cross-over to the diffusion regime is clearly manifested. In the cases $\mu_a/\mu_s = 1$ and 10 there is no such cross-over and the DA curves are not shown.

In conclusion, we have presented a general numerically efficient method for solving the RTE in an infinite macroscopically homogeneous medium. The method allows one to solve the RTE for arbitrary phase functions $A(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = f(\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}')$ by numerically diagonalizing several tridiagonal matrices. Once eigenvalues and eigenvalues of these matrices are found, the solution is obtained analytically. This fact, together with the relatively low computational complexity, distinguish the suggested method from other approaches. Boundary conditions and interfaces have not been discussed in this letter and will be considered elsewhere.

References

[1] van Rossum M C W and Nieuwenhuizen T M 1999 *Rev. Mod. Phys.* **71** 313

[2] Arridge S R 1999 *Inverse Problems* **15** R41

[3] Chandrasekhar S 1960 *Radiative Transfer* (New York: Dover)

[4] Sobolev V V 1976 *Light Scattering in Planetary Atmospheres* (*Int. Series of Monographs in Natural Philosophy*) (Oxford: Pergamon)

[5] Apresyan L A and Kravtsov Yu A 1996 *Radiative Transfer. Statistical and Wave Aspects* (Amsterdam: Gordon and Breach)

[6] Case K M and Zweifel P F 1967 *Linear Transport Theory* (Reading, MA: Addison-Wesley)

[7] Ishimaru A 1978 *Wave Propagation and Scattering in Random Media* (San Diego, CA: Academic)

[8] Hull E L and Foster T H 2001 *J. Opt. Soc. Am.* A **18** 584

[9] Mueller D W and Crosbie A L 2002 *J. Quant. Spectrosc. Radiat. Transfer* **72** 19

[10] Kim A D and Keller J B 2003 *J. Opt. Soc. Am.* A **20** 92